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Numerical investigation of fully developed laminar flow in irregular annular ducts: Triangular–circular combinations

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ABSTRACT

The aim of this study is to reduce the required pumping energy by obtaining accurately the friction factor – Reynolds number product (*fRe*) of the steady fully developed laminar flow in annular ducts. The study is focused on the annular region between equilateral triangular and circular ducts under all possible combinations. For this purpose, the governing equations are solved using high order finite element method. For regular annuli, it is found that higher values of area ratio lead to monotonic increase in (*fRe*) value, with $(fRe)_{max}$ = (24, 42.67,96) at the respective values of (D_h) = (0.5,0.75,1) regardless of the particular geometry. For irregular annuli, higher values of area ratio lead to an increase followed by a decrease in (*fRe*) value, with $(fRe)_{max}$ = (79.631,35.392,19.921) at the respective values of (D_h) = (0.5, 0.75, 1) for the (*CT*) case, and correspondingly $(fRe)_{max}$ = (91.02,40.45,22.85) for the (*TC*) case. On the other hand, it is found that (*fRe*) value inversely proportional with the hydraulic diameter (D_h). For all cases considered in this study, the largest (*fRe*) at the representative values (*AR*) = 30% is found for the (*CC*) case with $(fRe)_{max}$ = 95.43 whereas the smallest (*fRe*) is found for the (*CT*) case with $(fRe)_{min}$ = 17.544. More importantly, irregular annuli outperformed the regular annuli and thus are recommended to replace the classical regular annuli currently used in double duct heat exchangers. This in turn will significantly decrease the pumping energy required in such applications in industry.

1. Introduction

Analyzing laminar flow through single and annular ducts was the main target for many researchers due to the involvement of such flows in many engineering applications, such as piping systems and heat exchangers. Therefore, laminar flow in single duct was investigated thoroughly during the second half of the last century. One of the most comprehensive studies in this field was conducted by Sparrow [1], in which the product of friction factor and Reynolds number (*fRe*) values were obtained for the case of laminar flow in a wide range of single-duct cross-sectional shapes. Saha [2] obtained numerically the (*fRe*) values of three different single non-circular ducts (triangular, square, and sinusoidal duct). Similar work was carried out by Uzun and Unsal [3], and they found that (*fRe*) values obtained in their study are slightly higher than the values obtained by Saha [2].

Pendergast et al. [4] established analytical solution for the velocity profile of steady incompressible laminar fluid flow through ducts of arbitrary cross-sectional area and compared their analytical model with experimental data. The results showed that the new analytical model for velocity profile agreed significantly with the previous experimental data. Ray and Date [5] investigated the effect of inserting twisted tape inside a square duct. They studied the relationship between the friction factor (*f*) and Reynolds number (*Re*) and they obtained a correlation for (*f*) that showed excellent agreement with available experimental results. In addition, they showed that the friction factor is inversely proportional with Reynolds number. Xue [6] applied the Galerkin method to analyze the laminar flow in helical pipes. This study opened the gate for using the Galerkin method in the analysis of laminar flow in ducts.

Ray and Misra [7] investigated the effect of adding fillets to square and equilateral triangular ducts. In their work, they showed that the added fillet increases the (*fRe*) value for the small range of fillet radius, and decreases the (*fRe*) value when the range of fillet radius increases for both square and triangle ducts. This result also approved by Chakraborty and Ray [8] for the square ducts. Another comprehensive study in this field was also carried out by Muzychka and Yovanovich [9]. They studied the friction loss in ducts by focusing on the product value of friction factor and

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Nomenclature

A	cross sectional area of flow (m^2)
AR	cross-sectional area ratio "inner duct area to outer duct area"
CC	cylinder-inside-cylinder annulus
CT	cylinder-inside-triangle annulus
D_h	hydraulic diameter (m)
e	error
f	fanning friction factor
L_1	side length of the internal triangular duct (m)
L_2	side length of the external triangular duct (m)
m	degree of the interpolating polynomial
N	non-dimensional inward drawn normal
n	inward drawn normal
P	non-dimensional pressure
p	pressure (Pa)
r_1	radius of the internal duct (m)
r_2	radius of the external duct (m)
Re	Reynolds number
ST	square-inside-triangle annulus
TT	triangle-inside-triangle annulus
TC	triangle-inside-cylinder annulus
U, V, W	non-dimensional velocity components
u, v, w	velocity components in (x, y, z) directions, respectively (m/s)
U_{avg}	non-dimensional average velocity in the axial direction
u_{avg}	average velocity of flow in the axial direction (m/s)
x, y, z	Cartesian coordinates (m)
X, Y, Z	non-dimensional Cartesian coordinates

Subscripts

1	internal
2	external
h	hydraulic
w	duct wall
avg	average

Greek symbols

τ_w	mean wall shear stress (N/m^2)
ρ	density (kg/m^3)
μ	dynamic viscosity (Pa s)
ν	kinematic viscosity of fluid (m^2/s)

Reynolds number (*fRe*), considering the laminar flow in a wide range of single duct cross-sectional shapes.

The published work in the literature related to laminar flow in annulus can be divided into two main categories: regular and irregular annuli. Laminar flow in regular annuli was investigated intensively due to its relative simplicity in addition to its industrial importance (e.g. in heat exchangers). For both cases of concentric and eccentric regular annuli, many different shapes were considered. The circular annulus (Circular-inside-Circular ducts), which is considered as the basic shape, was the target of many studies due to its wide spread applications.

Lee and Kuo [10] studied the concentric case and focused on the evaluation of friction factor Reynolds number product (*fRe*), for different values of area ratio (inner to outer cylinders), using the finite volume method. They found that the more accurate solutions of (*fRe*) value obtained, the better results of Nusselt number are estimated. Also, the values of (*fRe*) when the aspect ratio between 0.6 and 0.9 and Nusselt number equals to 3 were in excellent agreement with those provided by Shah and London [17]. Furthermore, (*fRe*) value approached asymptotically to 24 when the aspect ratio was in the vicinity of unity. As the aspect ratio approached zero, i.e. single circular duct, (*fRe*) value was exactly 16 by using one eigenvalue.

Atayilmaz [11] presented values of the streamlines and isotherms for horizontal concentric cylinders theoretically and experimentally, for $Ra_L = 9 \times 10^5 - 5 \times 10^6$, and $Ra = 2 \times 10^5 - 7 \times 10^5$. He used the domain–free discretization method and found that the bottom region is essentially stagnant with low velocities and the temperature distribution is divided into hot and cold regions by an imaginary line below the inner cylinder. However, in the top half of the cylinder, the fluid is re-circulated making the outer layer warmer. Shu and Wu [12] extended their work and presented an efficient numerical approach, the domain-free discretization DFD method, to solve partial differential equations (PDEs) on a doubly connected domain. The proposed method was applied to simulate the natural convection in eccentric annuli. The vorticity-stream function formulation of the governing equations in cylindrical coordinates is used, and the pressure single-value condition is used to update the stream function value on the inner cylinder wall. It was found that the numerical results obtained by the DFD method agree well with the available data in the literature. The study concluded that the DFD method is a promising tool in dealing with complex physical domains.

Due to the increased need for higher heat transfer rates in industry, fully developed laminar flow in non-circular annuli drew more attention lately. Laminar flow in such annular configuration was investigated by Venkateswara et al. [13]. In their experimental work, three different concentric equilateral triangular annuli with different side lengths for the inner duct, and constant side length for the outer duct (i.e. three test sections) were tested. The results of their work showed that the experimental (*fRe*) values are 24, 48 and 26, while the corresponding values from the numerical analysis are 19.7, 20 and 21 for the three channels. This discrepancy was attributed to the sharp corners of the annulus.

Asan [14] studied the steady-state, laminar, two-dimensional natural convection in an annulus between two isothermal concentric square ducts. In this study, the stream function-vorticity formulation was applied and control volume integration solution technique was adopted. Solutions are obtained for Rayleigh number values up to 10^6. Three different dimension ratios are considered. The effects of dimension ratio and Rayleigh number on the flow structure and heat transfer were investigated. The results showed that the dimension ratio and Rayleigh number have a profound influence on the temperature and flow fields.

On the other hand, the irregular annuli suffered from lack of research effort due to the inherent complexity. However, because of the revolution taking place in the field of computer software during the past two decades, the trend now is toward the study of complex irregular annuli. Some of the recognized effort in this area was accomplished by Haddad and Al-Odat [15]. The aim of their work was to obtain accurate prediction of the friction factor-Reynolds number product (*fRe*) using high-order finite element method. It is found that higher area ratio will lead to monotonic increase in (*fRe*) value in the case of regular annuli, while leads to an increase followed by a decrease in (*fRe*) value in the case of irregular annuli. Among all possible arrangements of the square and triangular ducts, irregular annuli had lower (*fRe*) values than regular annuli, and the square-in-triangle annulus had the lowest (*fRe*) value, whereas the square-in-square annulus had the highest (*fRe*) value.