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A mathematical technique for hybrid power system design with energy loss considerations

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ABSTRACT

This paper presents a generic mathematical optimisation model for the design of hybrid power systems (HPSs). The model takes into account power losses during the allocation of power generated from renewables to appliance loads, and is formulated as a linear programme (LP) based on a superstructure including all possible power allocation options in a typical HPS. With given power source and demand data for an HPS, the minimum outsourced electricity supply and the minimum electricity storage capacity required can be determined through a two-step optimisation. Three literature case studies are solved to illustrate the proposed approach.

1. Introduction

Hybrid power systems (HPSs), as the name implies, involve the utilisation of two or more renewable and conventional energy sources for electricity generation [1,2]. Renewable energy (RE) sources (e.g. solar, wind and biomass) provide clean and sustainable power production and serve as potential alternatives to conventional sources. In spite of the intermittent and unpredictable nature of some RE sources, integrating complementary ones such as solar and wind can significantly improve the system efficiency and reliability, thereby reducing the dependence on backup energy devices (e.g. batteries, diesel generators and fuel cells) [3,4]. It is this complementary characteristic between solar and wind energy that makes such hybrid systems a popular topic attracting considerable research interest. The main focus has been given to the sizing and optimisation of HPSs [5–13]. Research efforts are also found in evaluating HPS performance [14] and optimal power generation and load management in HPSs [15]. In addition, HPSs are widely considered a viable option for electrifying remote areas where grid extension is not feasible or economical and fuel transportation is expensive [16–18].

Recently, the principles and tools of *process integration* (PI) have been extended to design and optimise HPSs. PI is defined as "a holistic approach to process design, retrofitting and operation which emphasizes the unity of the process" [19,20]. There are two general categories of PI techniques, namely *pinch analysis* and *mathematical optimisation*. The former sets performance targets ahead of detailed system design, and provides good insights into the design problem, while the latter is often preferred for handling complex problems with multiple quality indices or with operational or topological constraints. Essentially, HPS design is analogous to the *synthesis of batch resource conservation networks*, an active branch of PI [21–23].

Based on the design space approach, Sreeraj et al. [24] proposed a method to optimally size a renewable hybrid system and evaluate the cost of power generated. Their method takes into account uncertainties of power sources and incorporates a PI-based methodology to determine the minimum battery capacity required. The use of the *grand composite curve* representation of stored energy for isolated RE system sizing was later proposed by Bandyopadhyay [25]. Both methodologies assist in the planning of further load expansion without increasing the size of the system [24,25]. More recently, a pinch-based numerical technique known as the *electric system cascade analysis* was developed by Ho et al. [26] for designing and optimising isolated distributed energy generation systems. However, this technique is only applicable to solar-based HPSs, and not to wind or solar-wind systems. For the integration of solar thermal energy, a model was developed by Nemet et al. [27] to approximate the measured irradiation profile by minimising the number of time slices and the overall inaccuracy.

Wan Alwi et al. [28] established the *power pinch analysis* (PoPA) technique for optimal power allocation in HPSs, and introduced a graphical PoPA tool termed the *power composite curves* to

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Nomenclature

Indices and sets

$i \in \mathbf{I}$	power sources
$i \in \mathbf{I}^{ac} \subseteq \mathbf{I}$	AC power sources
$i \in \mathbf{I}^{dc} \subseteq \mathbf{I}$	DC power sources
$j \in \mathbf{J}$	power demands
$j \in \mathbf{J}^{ac} \subseteq \mathbf{J}$	AC power demands
$j \in \mathbf{J}^{dc} \subseteq \mathbf{J}$	DC power demands
$s \in \mathbf{S}$	electricity storage systems
$t \in \mathbf{T}$	time intervals

Parameters

P_{it}	rated power generation from source i in time interval t (kW)
P_{jt}	power rating of demand j in time interval t (kW)
Q_s^{cap}	capacity of storage system s (kW h)
Z^{op}	binary parameter associated with the state of HPS operation (dimensionless)
Δ_t	length of time interval t (h)
η^{inv}	inverter (DC–AC) efficiency (dimensionless)
η^{rec}	rectifier (AC–DC) efficiency (dimensionless)
η_s^{in}	charging efficiency of storage system s (dimensionless)
η_s^{out}	discharging efficiency of storage system s (dimensionless)
σ_s	hourly self-discharge rate of storage system s (1/h)

Variables

p_{ijt}	power from source i to demand j in time interval t (kW)
p_{ist}	power from source i to storage system s in time interval t (kW)
p_{it}^{w}	waste power from source i in time interval t (kW)
p_{jt}^{os}	outsourced power to demand j in time interval t (kW)
p_{sjt}	power from storage system s to demand j in time interval t (kW)
p_{st}^{in}	power charged into storage system s in time interval t (kW)
p_{st}^{out}	power discharged from storage system s in time interval t (kW)
q_{st}	electricity stored in storage system s at the end of time interval t (kW h)

determine the minimum outsourced electricity supply and the amount of excess electricity during start up and normal operation. Two numerical PoPA tools termed the *power cascade table* and the *storage cascade table* (SCT) were later developed by Mohammad Rozali et al. [29] to provide the maximum power demand and the required electricity storage capacity in addition to the electricity targets. However, these studies assume 100% power transfer and battery storage efficiency, an ideal state that cannot be achieved in practice. To overcome this drawback and improve the PoPA accuracy, Mohammad Rozali et al. [30] further developed the SCT by considering power losses during the conversion, transfer and storage of power. The modified SCT technique is based on prioritised options for power allocation; however, this sequential nature may lead to suboptimal results. Furthermore, the calculations for constructing the modified SCT could be cumbersome when the number of time intervals increases. Most recently, Chen et al. [31,32] presented two transshipment model-based mathematical formulations for the targeting and design of HPSs. Their formulations account for the effect of power losses from transfer and storage by including a fixed discount ratio, although not considering the conversion losses. Arguably, the adoption of the transshipment model for HPS design tends to complicate the problem formulation.

In this paper, a superstructure-based mathematical model will be developed for HPS design with power loss considerations. The superstructure approach makes the model generic and flexible to accommodate further design options or constraints. Furthermore, the mathematical programming framework allows all the power allocation options to be considered simultaneously and hence the true optimum to be located. In the following sections, a formal problem statement is given first. The superstructure and the mathematical model are presented next. Three illustrative case studies are then used to demonstrate the application of the proposed model.

2. Problem statement

The problem addressed in this paper can be formally stated as follows. A typical HPS consists of a set of power sources $i \in \mathbf{I}$ and a set of power demands $j \in \mathbf{J}$. Power sources are renewables such as solar and wind to produce power for demands from various appliances. The available/operating times of sources and demands are given, as well as their power ratings. In view of the intermittent behaviour of power sources and demands, a set of electricity storage systems $s \in \mathbf{S}$ are to be used to facilitate power allocation. When there is insufficient power to satisfy the demands, the deficit is made up by outsourcing electricity from the grid (or alternatively by onsite generation). Since power produced from sources and the demand load can be alternating (AC) or direct current (DC), a power conditioning system is needed. Specifically, the conversion between AC and DC involves the rectifier (AC–DC) and the inverter (DC–AC). It is assumed in this work that the set of storage systems is comprised of DC batteries. Additionally, the outsourced electricity is assumed to be AC. The objective is to determine the minimum outsourced electricity supply and the minimum electricity storage capacity achieving this target.

3. Superstructure and mathematical model

In this work, the modelling of an HPS is based on a superstructure consisting of three key HPS elements, namely power sources (Fig. 1a), power demands (Fig. 1b) and electricity storage systems (Fig. 1c). This superstructure includes all feasible options for power allocation in typical HPSs. To address temporal issues arising from intermittent power sources and demands, of which the timings may not fully coincide, a set of time intervals $t \in \mathbf{T}$ are defined according to the source and demand data. The mathematical model presented below consists mainly of energy balance equations. Notation used is given in the Nomenclature.

Fig. 1a shows a schematic diagram of a power source. The power generated from source i can be supplied immediately to power demands j, stored in electricity storage systems s for later use, or dumped as waste power in case of excessive power generation. Eq. (1) describes the energy balance for source i in time interval t:

$$P_{it} = \sum_{j \in \mathbf{J}} p_{ijt} + \sum_{s \in \mathbf{S}} p_{ist} + p_{it}^{w} \quad i \in \mathbf{I}, t \in \mathbf{T} \tag{1}$$

Fig. 1b shows a schematic diagram of a power demand. The power supplied to demand j may come from power sources i, electricity storage systems s or the grid (i.e. outsourced power). Eqs. (2a) and (2b) describe the energy balances for AC ($j \in \mathbf{J}^{ac}$)