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Microstepping and high-performance control of permanent-magnet stepper motors

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ABSTRACT

We examine the problem of control of high-performance drives and servos with permanent-magnet stepper motors. Control of electromechanical systems implies control and optimization of electromechanical transductions and energy conversion. Robust spatio-temporal control algorithms are designed to ensure high efficiency, high-precision microstepping and optimal performance. The system stability, robustness and control design are examined applying an *admissibility* concept. Nonlinear control guarantees optimal energy conversion in expanded operating envelopes. Our analytic designs are substantiated and verified. A proof-of-concept system is tested and characterized. The high electromagnetic torque and high-precision microstep angular positioning simplify kinematics, enables efficiency, ensures direct-drive capabilities, reduces complexity, etc. For four-phase permanent magnet stepper motors, one may ensure up to 256 microsteps within a 1.8° full step. High efficiency and accurate 2.454×10^{-4} rad positioning (25,600 microsteps per revolution) are achieved with high electromagnetic and holding torques. To guarantee high efficiency, optimality and enabled energy conversion capabilities, electromechanical energy conversion and high electromagnetic torque are achieved by applying *soft* balanced phase voltages. The ripple and friction torques are minimized. The fundamental findings, technology-centric design and experimental results are reported.

1. Introduction

Fundamental, applied and practical problems of optimal energy conversion and precision motion control of electromechanical systems are of a great importance. The solution of the aforementioned problems allow one to maximize the efficiency, minimize losses, as well as deploy advanced motion platforms in aerospace, automotive, electronics, energy, manufacturing, power, robotics and other applications. Enabling technologies are under developments. Permanent-magnet stepper motors are widely used in drives and servos due to gearless direct-drive capabilities. Stepper motors are simple, affordable, rugged. Performance and capabilities may be enabled applying recent fundamental and technological advancements ensuring high torque and high power densities. The specified dynamics and high precision microstepping can be achieved developing high electromagnetic torque. Optimal electromagnetic and electromechanical designs, rear-earth permanent magnets and advanced materials enable energy conversion capabilities. The technology-enhanced stepper motors are controlled by advanced power electronics. Consistent control schemes ensure optimal energy conversion which results in enabled functionality and performance. Coherent device- and system-level solutions imply the use of advanced software, electronics and electromechanical hardware. New trends in motion control of drives and servos foster developments of nonlinear control algorithms consistent with the device electromagnetics and power electronics [1–3]. *Sensorless* and sensor-centric designs of servos with stepper motors are reported in [4–8] using the *quadrature* and *direct* voltages and currents. However, electric machines are controlled changing the phase voltages [1–3,9–12]. Practical concepts in motion control of permanent-magnet stepper motors were developed for full-, half- and quarter stepping. An optimal energy conversion and high-precision microstepping implies the use of advanced-technology motors, electronics and control schemes.

Many studies concentrate on modeling [7,8]. The use of the *arbitrary* reference frame and *quadrature-direct* quantities serve as a possible inroad to stability analysis and design of *feedback linearizing* and *vector* controls [4–8]. While analytic results may be important, practical control schemes use the electromagnetically-consistent

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machine variables [1–3,9–12]. The directly measured phase currents and voltages are used as physically-consistent controllable variables. Due to the algorithmic, software and hardware complexity, it is difficult to implement the spatio-temporal motion control using the mathematically-descriptive *quadrature*- and *direct*-components of voltages, fluxes and currents. Optimal energy conversion in stepper motors requires electromagnetically-consistent concepts. The use of adaptive, artificial neural networks, *intelligent*, *sensorless* and other control schemes may not ensure adequate energy conversion. Practical solutions require technology-centric fundamental research. These transformative findings result in new knowledge generation and developments which enable engineering and technology.

2. Electromagnetics and electromechanics of permanent-magnet stepper motors

By using the laws of electromagnetics and electromechanics, the differential equations for two- and four-phase permanent-magnet stepper motors in the *machine* variables are derived [1,2]. For two-phase motor, the *a* and *b* flux linkages are $\psi_a = Li_a + \psi_{am}$ and $\psi_b = Li_b + \psi_{bm}$. Assuming a sinusoidal uniform magnetic field of permanent magnets, one has $\psi_{am} = \psi_m\cos(RT\theta_m)$ and $\psi_{bm} = \psi_m\sin(RT\theta_m)$. We obtain [1,2]

$$\frac{di_a}{dt} = \frac{1}{L}[-ri_a + RT\psi_m\omega_m\sin(RT\theta_m) + u_a],$$

$$\frac{di_b}{dt} = \frac{1}{L}[-ri_b - RT\psi_m\omega_m\cos(RT\theta_m) + u_b],$$

$$\frac{d\omega_m}{dt} = \frac{1}{J}[RT\psi_m(-i_a\sin(RT\theta_m) + i_b\cos(RT\theta_m)) - B_m\omega_m - T_L],$$

$$\frac{d\theta_m}{dt} = \omega_m, \tag{1}$$

where i_a and i_b are the *a* and *b* phase currents; u_a and u_b are the applied phase voltages to the *a* and *b* windings; ω_m and θ_m are the mechanical angular velocity and rotor displacement; T_L is the load torque; *r* and *L* are the resistance and self-inductance of the stator winding; *RT* is the number of rotor teeth per stack; ψ_m is the amplitude of the flux linkages established by the permanent magnet; *J* is the equivalent moment of inertia; B_m is the viscous friction coefficient.

The expression for the electromagnetic torque is

$$T_e = RT\psi_m[-i_a\sin(RT\theta_m) + i_b\cos(RT\theta_m)]. \tag{2}$$

The stator resistance *r*, self-inductance *L*, amplitude the flux linkages ψ_m, viscous friction coefficient B_m, and equivalent moment of inertia *J* vary due to varying temperature and loads. These variations are bounded, and, $r \in [r_{\min}\ r_{\max}]$, $L \in [L_{\min}\ L_{\max}]$, $\psi_m \in [\psi_{m\min}\ \psi_{m\max}]$, $B_m \in [B_{m\min}\ B_{m\max}]$ and $J \in [J_{\min} J_{\max}]$. These bounded parameters $a_{i\min} \leqslant a_i(\cdot) \leqslant a_{i\max}$ are always positive, $a_i(\cdot)>0$, $a_{i\min} > 0$ and $a_{i\max} > 0$.

To perform the stability analysis, we use the *arbitrary* reference frame which rotates at synchronous angular velocity [1,2]. The Park transformation yields the *quadrature*- and *direct*-axis components of currents $\begin{bmatrix} i_q^r \\ i_d^r \end{bmatrix} = \begin{bmatrix} -\sin(RT\theta_m) & \cos(RT\theta_m) \\ \cos(RT\theta_m) & \sin(RT\theta_m) \end{bmatrix}\begin{bmatrix} i_a \\ i_b \end{bmatrix}$ and voltages $\begin{bmatrix} u_q^r \\ u_d^r \end{bmatrix} = \begin{bmatrix} -\sin(RT\theta_m) & \cos(RT\theta_m) \\ \cos(RT\theta_m) & \sin(RT\theta_m) \end{bmatrix}\begin{bmatrix} u_a \\ u_b \end{bmatrix}$. Using (1) and (2), one obtains [1,2]

$$\frac{di_q^r}{dt} = \frac{1}{L}\left[-ri_q^r - RT\psi_m\omega_m - LRTi_d^r\omega_m + u_q^r\right],$$

$$\frac{di_d^r}{dt} = \frac{1}{L}\left[-ri_d^r + LRTi_q^r\omega_m + u_d^r\right],$$

$$\frac{d\omega_m}{dt} = \frac{1}{J}\left[RT\psi_m i_q^r - B_m\omega_m - T_L\right]. \tag{3}$$

For a positive-definite function $V\left(i_q^r, i_d^r, \omega_m\right) = \frac{1}{2}\left(i_q^{r2} + i_d^{r2} + \omega_m^2\right)$, the total derivative is $\frac{dV\left(i_q^r, i_d^r, \omega_m\right)}{dt} = \frac{1}{L}\left[-ri_q^{r2} + \frac{RT\psi_m(L-J)}{J}i_q^r\omega_m + i_q^r u_q^r - ri_d^{r2} + i_d^r u_d^r - B_m\omega_m^2\right]$. Therefore, there exists a positive-definite Lyapunov function $V\left(i_q^r, i_d^r, \omega_m\right) > 0$, for which $\frac{dV\left(i_q^r, i_d^r, \omega_m\right)}{dt} < 0$. That is, dV/dt is negative-definite for any variations of bounded motor parameters $a_i(\cdot)$, $a_{i\min} \leqslant a_i(\cdot) \leqslant a_{i\max}$, $a_{i\min} > 0$, $a_{i\max} > 0$. Hence, the open-loop permanent-magnet stepper motor is robustly asymptotically stable in the large.

The high-fidelity mathematical models and expressions for the electromagnetic torque T_e are derived by using a consistent expression for the flux linkages. A nonlinear electromagnetic system, nonuniform and nonstationary magnetic field of permanent magnets and other phenomena result in the equations for $\psi_a(\theta_m)$ and $\psi_b(\theta_m)$ as

$$\begin{bmatrix} \psi_a \\ \psi_b \end{bmatrix} = \begin{bmatrix} L & 0 \\ 0 & L \end{bmatrix}\begin{bmatrix} i_a \\ i_b \end{bmatrix} + \psi_m \begin{bmatrix} \sum_{n=1}^{\infty} b_n\cos^{2n-1}(RT\theta_m) \\ \sum_{n=1}^{\infty} b_n\sin^{2n-1}(RT\theta_m) \end{bmatrix}, \tag{4}$$

where b_n are the positive coefficients which describe the nonlinear electromagnetics. For example, for the ideal sinusoidal electromagnetic coupling, ensured by the Aerotech, Kollmorgen, Shinano Kenshi and other permanent-magnet synchronous machines, $b_1 = 1$ and $\forall b_n = 0$ for $\forall n > 1$ $(n = 2,3,4,\ldots)$. For high electromagnetic loadings, inadequate design and gaps (spacing) between magnets, $\forall b_n \neq 0$ with $b_1 > b_2 > b_3\ldots$

The electromagnetic system, electromagnetic loadings and other effects affect T_e. From (4), the expression for the electromagnetic torque is

$$T_e = RT\psi_m\left[-i_a\sum_{n=1}^{\infty}(2n-1)b_n\sin(RT\theta_m)\cos^{2n-2}(RT\theta_m) + i_b\sum_{n=1}^{\infty}(2n-1)b_n\cos(RT\theta_m)\sin^{2n-2}(RT\theta_m)\right]. \tag{5}$$

Using the expressions for the electromagnetic torque (2) and (5), one finds the balanced current and voltage sets which maximize the electromagnetic torque T_e and minimize the torque ripple. From (2), the balanced current and voltage sets are derived for: (i) Full step operation with the angular displacement $2\pi/RT$, π/RT or $\pi/2RT$, which depends on the motor and windings designs; (ii) Continuous motor rotation. In particular, we have

$$\begin{aligned} &i_a = -i_M\sin(RT\theta_m), i_b = i_M\cos(RT\theta_m), \quad i_{M\min} \le i_M \le i_{M\max}, \\ &u_a = -u_M\sin(RT\theta_m), u_b = u_M\cos(RT\theta_m), u_{M\min} \le u_M \le u_{M\max}, \end{aligned} \tag{6}$$

where i_M and u_M are the rated current and voltage.

The balanced current and voltage sets (6) suit the operation of stepper motors in the full operating envelope including high electromagnetic loadings. The derived expressions for T_e yield the balanced fed phase currents and applied voltages which ensure the full-, half-, quarter- and microstepping [1,2,12,13]. Assuming $2\pi/RT$ displacement for a full step, using (2) and (5), one has