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Convection-radiation thermal analysis of triangular porous fins with temperature-dependent thermal conductivity by DTM

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Increasing performance of porous fins is one of the common priorities nowadays. Hence attention here is given to the convection and radiation effects in the analysis of performance of a porous triangular fin with temperature-dependent thermal conductivity. Mathematical problem is solved by differential transformation method (DTM). The DTM results are compared with the numerical results obtained by fourth-order Runge–Kutta method in order to confirm the accuracy of the analytical solution. The dimensionless temperature distributions, fin efficiency and fin effectiveness are studied with respect to emerging parameters involved in the analysis. It is noted that there is an increase in fin performance in view of porous constraint.

1. Introduction

Heat transfer has a pivotal role in various engineering problems. More, the fins or extended surfaces are important in enhancement of the rate of heat transfer from the primary surface. The profile of a fin is generally selected on the basis of the cost of material and manufacturing as well as on the ease of fabrication. There is a wide use of rectangular fins in such situations. This perhaps is related to ease in its design and manufacturing. The attempts highlighting the analysis of fin performance may be mentioned in the studies [1–7]. In these researches, the optimum profile has also been determined to obtain maximum rate of heat transfer. A triangular fin due to reducing volume (fin material) than a rectangular fin for an equal heat transfer is attractive. Thermal analysis of a viscoelastic fluid past a triangular fin was numerically studied by Hsiao and Hsu [8]. They found that the elastic effect in the flow can increase local heat transfer and enhance the heat transfer of a triangular fin. Kundu et al. [9] presented an analytical method for predicting fin performance of triangular fins subject to simultaneous heat and mass transfer. They observed that the performance of wet fins is almost independent on the relative humidity. Performance analysis and optimization of straight taper fins with variable heat transfer coefficient was analytically studied by Kundu and Das [10]. They applied the Frobenius series method to obtain temperature profiles in longitudinal fin.

There is an increasing interest of the recent researchers through heat transfer in porous media for increasing heat transfer rate of fins. Increasing the effectiveness area through which heat is convected to surrounding fluid is an original mechanism just to improve heat transfer using porous fins. Thermal analysis of porous fin for increasing heat transfer rate from a given surface was numerically investigated by Kiwan and Al-Nimr [11]. Kiwan [12] analyzed the performance of porous fins in natural convection environment. Gorla and Bakier [13] numerically investigated the thermal analysis of natural convection and radiation in a rectangular porous fin. They concluded that porous fins provide much higher heat transfer rate than the conventional solid fins. An analytical model for determination of the performance and optimization of porous fins with respect to the different models of prediction was presented by Kundu and Bhanja [14]. MHD effect on a rectangular porous fin was investigated by Taklifi et al. [15]. They concluded that by imposing MHD in system except near the fin tip, heat transfer rate from the porous fin decreases. Bhanja and Kundu [16] analytically investigated thermal analysis of a constructal T-shape porous fin with radiation effects. An increase in heat transfer is found by choosing porous medium condition in the fin.

Thermal radiation effects are quite significant in various engineering and industrial processes especially in the design of reliable equipments, nuclear plants, gas turbines and various propulsion devices for aircrafts, missiles, satellites and space vehicles. The

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Nomenclature

b	width of fin
C_p	specific heat of air
h	convection heat transfer coefficient
K	permeability of the porous fin
k	thermal conductivity
k_r	thermal conductivity ratio, k^*/k_f
k_1, k_2	constants in Eq. (16)
k^*	constant in Eq. (13)
L	length of the fin
$\dot{m}$	mass flow rate
q	actual heat transfer
Ra^*	modified Rayleigh number, $Ra^* = \frac{g\beta KLT_b}{\alpha\upsilon k_r\psi}$
Rd	radiation parameter, $Rd = \frac{\sigma\varepsilon T_b^3 L^2}{k^* y_b}$
T	temperature
υ_w	average velocity of the fluid passing through the fin at any point
x,y	coordinate starting from the fin tip
X	dimensionless coordinate
Z	fin parameter, $\sqrt{hL^2/k^* y_b}$
Greek symbols	
α	thermal diffusivity
β	thermal expansion coefficient
β^*	constant in Eq. (14)
β_R	Rosseland extinction coefficient
ε	emissivity
ε^*	fin effectiveness
ϕ	fin tip temperature
φ	porosity
γ	constant in Eq. (13)
η	fin efficiency
λ	ratio of semi-fin thickness at the base to width, y_b/b
θ	dimensionless temperature, $(T - T_a)/(T_b - T_a)$
ρ	density
σ	Stefan–Boltzmann constant
υ	kinematic viscosity
ω	dimensionless parameter, $1/2\lambda$
ψ	dimensionless semi-thickness at the base, y_b/L
Subscripts	
a	ambient
b	fin base
eff	effective properties
f	fluid

concept of thermal radiation is especially important in processes involving high temperature. The optimum dimensions of convecting-radiating fins was analytically determined by Razelos and Kakatsios [17]. Determination and measurement of the temperature along a fin cooled by natural convection and radiation was numerically and experimentally performed by Mueller and Abu-Mulaweh [18]. They observed that the heat loss due to radiation is typically 15–20% of total. Kiwan [19] analytically studied the effect of radiation on the heat transfer from porous fins. He observed that the radiation heat transfer becomes important as the surface temperature parameters increase, which the radiation effect becomes less important when the Rayleigh number increases. Heat transfer analysis from a horizontal fin array by natural convection and radiation was studied by Rao et al. [20] with the alternating direction implicit (ADI) method. Aziz and Green [21] analytically investigated performance and optimum design of convective-radiative rectangular fin with convective base heating, wall conduction resistant and contact resistant between the wall and the fin base. An approximate analytical solution for convection-radiation from a continuously moving fin with temperature-dependent thermal conductivity was developed by Aziz and Khani [22]. They employed the homotopy analysis method [HAM] for the development of solution. They concluded the radiative cooling becomes more effective for stronger radiation and consequently this lowers the temperature in the fin.

In this paper, the differential transform method (DTM) is employed for solving the nonlinear energy equation of a triangular porous fin with temperature-dependent thermal conductivity. It is evident from the existing literature that the convection-radiation heat transfer in triangular porous fins has not been pursued analytically. As mentioned above, Kundu et al. [9] just developed an analytical model via the Adomain decomposition method (ADM) to solve energy equation for a triangular wet fin without radiation and porosity effects. In present study, after obtaining explicit expression of temperature profiles, the efficiency of fin and fin effectiveness for different values of emerging parameters are also obtained and discussed. The temperature distributions obtained from the present model are compared with the numerical results obtained by fourth-order Runge–Kutta method. The concept of differential transformation method (DTM) was originally proposed by Zhou [23] in 1986 and it was used to solve both linear and nonlinear initial value problems in electric circuit analysis. The main benefit of this method is that it can be applied for both linear and nonlinear initial and boundary value problems without the need of linearization, discretization, or perturbation. DTM has been already applied for solution of both linear and nonlinear differential equations by many researchers [24,25]. Joneidi et al. [26] applied DTM to solve heat transfer problem from straight fin with temperature-dependent thermal conductivity. An analytical approach for predicting fin performance of three different fin profiles was conducted by Moradi and Ahmadikia [27] using DTM. Franco [28] analytically investigated optimum thermal design of convection longitudinal fin arrays via the differential transformation method. In present investigation, the obtained results are plotted and analyzed for these parameters.

2. Basic idea of DTM

The DTM is based on the Taylor series expansion and is a useful tool to develop analytical solutions of the differential equations. In this method, the governing differential equations and the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transforms of the original functions. The solution of these algebraic equations gives the desired solution of the problem.

Consider a function $f(x)$ which is analytic in a domain D and let $x = x_0$ represents any point in D. The function $f(x)$ is then represented by a power series whose center is located at x_0.The differential transform of the function $f(x)$ is

$$F(k) = \frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{1}$$

in which $f(x)$ is an original function and $F(k)$ is the transformed function. The inverse transformation is given by: