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Predicting geometry of rectangular and hyperbolic fin profiles with temperature-dependent thermal properties using decomposition and evolutionary methods

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ABSTRACT

This work proposes the application of the Adomian decomposition method (ADM) in conjunction with the differential evolution (DE) for simultaneously estimating the dimensions of a rectangular and hyperbolic profile annular fin in order to satisfy a prescribed temperature requirement. The thermal conductivity and the surface heat transfer are assumed to be temperature-dependent. The required temperature field has been obtained using ADM for cases, involving insulated and convective boundary conditions at the tip. Then, using an inverse scheme based on DE, required fins dimensions satisfying a prescribed temperature field are estimated. Owing to the correlated nature of the unknowns, many feasible solutions have been found to lie within a given range satisfying the given temperature field. This temperature field can offer the flexibility in selecting the designing parameters. The present study is expected to be useful for selecting the dimensions of a rectangular and hyperbolic profile annular fin which can satisfy the given temperature field.

1. Introduction

A fin is an extended surface used for increasing the heat transfer rate and possesses many useful applications [1]. In general, some heat is being conducted within the fin material and rest is lost from the surface of the fin. This heat transfer process depends upon the thermal conductivity of the fin material and the surface heat transfer coefficient, both of which may be temperature-dependent [2-4]. The variation of the thermal conductivity and the heat transfer coefficient with temperature incurs considerable nonlinearity in the governing energy equation. The general objective is to obtain the local thermal fields using either numerical or experimental analysis with *a priori* knowledge of the thermal properties and their dependency on the temperature and boundary conditions. This involves solution of a forward problem, which is mathematically well-posed. Many established techniques are available for solving problems of such kind [5, 6]. However, the situation becomes different and relatively more complex when the final objective in the form of temperature field is only available, but multiple parameters satisfying the given field remain unknown. Such problems are known as inverse problems [7], which are mathematically ill-posed and in conjunction with a forward algorithm, they require either some optimization or regularization methods [8]. The inverse problems are generally design-oriented and a given objective may have multiple solutions depending upon the nature of the unknown parameters.

The inverse problems and optimization studies involving heat transfer through fins have been one of the subjects of recent research. For example, Huang et al. [9] and Chen et al. [10] have respectively estimated the conductance in a finned-tube heat exchanger and the heat transfer coefficient of an annular fin using the conjugate gradient method (CGM). In a rectangular fin, the unknown boundary conditions satisfying a given temperature history were estimated by Abboudi [11] using an iterative regularization and CGM. Das [12] estimated the convective–conductive parameter and the variable conductivity parameter in a rectangular fin using the simplex search method. Liu [13] used a fuzzy approach for designing longitudinal fin arrays. Recently, Azimi et al. [14] have predicted the base temperature in non-Fourier fins using the Adjoint CGM. The surface heat transfer coefficient in finned heat exchangers using the steepest descent method was used by Huang and Tsai [15]. Xie et al. [16] have applied the genetic algorithm (GA) for designing and optimizing fin-and-tube heat exchangers in order to estimate the thermal conductivity and the surface heat transfer coefficient in cylindrical fin geometry. Das [17] also implemented GA to estimate the unknown parameters in cylindrical fin. Rao and Patel [18] demonstrated the application of particle swarm optimization method for cross flow plate-fin heat exchanger. The

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Nomenclature

A_c	cross sectional area of rectangular fin, m^2
A_r	cross sectional area of hyperbolic fin, m^2
A_s	local surface area of hyperbolic fin, m^2
A_m, B_m, C_m, D_m, E_m, F_m	Adomian polynomials
Bi	Biot number ($=hL/k_a$)
c	unknown fin tip temperature (in Eq. (16))
$c_{i,j}$	integral constants (in Eqs. (20) and (28))
CR	crossover ratio
e_r	measurement error
F	objective function
h	heat transfer coefficient, W/m^2 K
k	thermal conductivity, W/m K
L	length of fin, m
M	dimensionless fin parameter for hyperbolic fin, $\left(=\sqrt{hr_0^2/k_a y_b}\right)$
M_p	population vector of differential evolution (DE)
N	dimensionless fin parameter for rectangular fin, $\left(=\sqrt{hPL^2/k_a A_c}=\sqrt{Bi/\psi}\right)$
n	exponent for variable heat transfer coefficient
P	perimeter of the fin, m
R	dimensionless radius ($=r/r_0$)
r	local radius of hyperbolic profile annular fin, m
$rand$	random number
S	scaling factor
t	fin thickness, m
T	temperature, K
w	fin width, m
x	dimensional fin length, m
y	semi-fin thickness, m

Greek symbols

β	variable conductivity parameter
λ	coefficient of variable thermal conductivity, 1/K
θ	non-dimensional temperature, $\theta=\left(\frac{T-T_a}{T_b-T_a}\right)$
ϕ_x, ϕ_R	terms arising from integration
ξ_x, ξ_R	linear operator of Adomian decomposition method
ξ_x^{-1}, ξ_R^{-1}	inverse operator of Adomian decomposition method
ψ	fin aspect ratio ($t/2L$)

Subscripts

a	ambient
b	fin base
$conv$	convective
k	index for a particular member in the population
$insul$	insulated
0	tip of the hyperbolic fin

Superscripts

$*$	index for non-dimensional parameters
$\sim$	exact value
$'$	derivative term

Levenberg–Marquardt algorithm for nonlinear parameter estimation in finned-tube coil evaporators was used by Bueno et al. [19]. For designing a porous fin, the simulated annealing was used by Das and Ooi [20]. The efficiency and design parameters of the rectangular wet fin are optimized by Sharqawy and Zubair [21] for various insulated and convective tip boundary conditions. Similarly, Sharqawy and Zubair [22] optimized the efficiency and design parameters of straight fin of different fin profile subjected to combined heat and mass transfer using an analytical solution.

It is well-known that for an objective function consisting of discontinuities and nonlinearities, the evolutionary search techniques outperform the conventional deterministic methods [23]. As a result, the present work has been carried out using evolutionary search method, i.e. DE, because of the existing non-linearity in the problem. The aim is to estimate the important dimensions such as perimeter, length and cross sectional area of a rectangular fin which will satisfy a prescribed temperature field, corresponding to temperature-dependent thermal conductivity and heat transfer coefficient. For the problem under consideration, the inherent difficulties involve nonlinearities due to variation of heat transfer coefficient and thermal conductivity along with the existence of correlated unknown design parameters. The objective of the present work is to successfully apply ADM and DE to estimate various feasible dimensions of a fin which will enable the designer to select the optimum configuration. Because of the simplicity and ease of manufacturing, the rectangular fin is considered for the present analysis and the same approach can be extended for other fin profiles as well.

It has been found from the literature that ADM-DE has not yet been implemented in inverse problems. Therefore, the present work is aimed at solving the nonlinear energy equation for a rectangular and hyperbolic profile annular fin using the ADM [24,25]. Whereas, the unknown design parameters are simultaneously estimated using the differential evolution (DE) algorithm [26], which is an inverse problem. ADM being an efficient numerical method for solving nonlinear equations has gained considerable attention [27,28], and the same has been used in this work. The differential evolution method is a robust and simple global search optimization method found to work well for nonlinear and multimodal objective functions [26,29,30]. It possesses good convergence and allows easy parallelization [29] as well. The following section describes the formulation and solution procedure of the presented problems followed by the results and discussions.

2. Formulation

A straight fin can be classified based on the fin surface profile. The variation of fin thickness along its length allows us to adjust the fin profile as rectangular, triangular, parabolic, etc. [22]. The relation for varying thickness along the length is given by [22] as

$$t_x = t_b(x/L)^j \tag{1}$$

where t_b is the fin thickness at the base and the exponent j is for adjusting different fin profiles. For a rectangular straight fin, $j = 0$, and thus the thickness remains uniform, i.e., $t_x = t_b$. Similarly, for triangular, convex parabolic and concave parabolic, the exponents are $j = 1$, $j = 1/2$ and $j = 2$, respectively. Thus, in case of triangular, convex parabolic and concave parabolic fin profiles, the thickness along the length varies, as $t_x = t_b\ (x/L)$, $t_x = t_b\ (x/L)^{1/2}$ and $t_x = t_b\ (x/L)^2$, respectively. A detail formulation and heat transfer analysis on various forms of fin profile are given by Sharqawy and Zubair [22]. In this work, at first the formulations of rectangular fin with two classes of boundary conditions, viz., adiabatic and convective at the fin tip has been studied. Next, the work presents the formulation of a different fin profile, i.e., hyperbolic profile annular fin subjected to adiabatic condition at the tip only.

2.1. Rectangular straight fin

A straight rectangular fin with uniform thickness, i.e., $j = 0$ and $t_x = t_b$ (Eq. (1)), temperature-dependent thermal conductivity, k