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Crack growth modeling in spur gear tooth and its effect on mesh stiffness using extended finite element method

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ABSTRACT

The manuscript is devoted to determine the time variable mesh stiffness (TVMS) and crack propagation behaviour in spur gear tooth using extended finite element method (XFEM). The TVMS of gear pairs provides significant information about the condition of the gear system. Thus, the accurate estimation of TVMS is essential to get the information of the primary source of vibration and noise. Extended finite element method is very effective for discontinuity due to faults like cracks etc. The capability of XFEM to enrich approximate spaces locally makes it appropriate method to compute the values of TVMS in spur gear. A gear tooth crack problem has been solved using XFEM and the obtained results are compared with the analytical results for validation. Further, a problem of crack at the root resign in thin rim gear is simulated to investigate the effect of rim thickness on gear tooth crack propagation path. The estimated crack propagation path shows good agreement with the results available in literature.

1. Introduction

Gearboxes are one of the most critical and widely used power transmission components in applications such as industrial machinery, automotive and aerospace. Due to their increasing applications and demand, health monitoring of gearbox is essential to avoid the abrupt failure of the system. The gear tooth failure occurs due to removal and/or plastic deformations on the contacting tooth surfaces or due to the presence of fatigue crack. The severity of tooth damage is usually measured in terms of reduction of the mesh stiffness [1]. Studies had been carried out to estimate time-varying gear mesh stiffness for improving the design and performance of the gears [1–3]. The time-varying mesh stiffness (TVMS) acts as an indicator of several factors affecting the condition of gear tooth engagement, and one of these factors is the existence of a fault in the teeth. An extreme stress concentration in an engaged tooth yields to a crack nucleation and propagation during the operation, which leads to the reduction of mesh stiffness value.

Early detection of a crack in spur gear allows proper scheduled shutdown of the system to prevent the catastrophic failure and consequently results in a safer operation and higher cost savings [1]. A crack may appear in gear tooth under severe operating conditions, such as excessive service load, inappropriate operating conditions, or simply fatigue [1, 2]. Zouari et al. [3] performed a three dimensional finite element analysis of spur gear to observe the effect of crack size, position and direction on the spectrum of the gear mesh stiffness. They found that the instantaneous mesh stiffness is affected significantly due to the presence of a crack in one or several teeth. Once a crack is nucleated, it is important to know that whether it will propagate through the tooth or through the rim. This information is required to avoid the failure and to maintain the performance of the gear system. Lewicki [4] found that the crack propagation path depends on the backup ratio which is defined as the ratio of rim thickness to tooth height. He observed that the gear

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tooth root crack propagates through the tooth along tooth width in a gear with a high backup ratio, while it propagates through the rim for a low backup ratio. The initial crack angle is also a crucial factor for the crack propagation. For the low initial crack angle, the crack propagation is through the rim even with the high backup ratio. Finite element method (FEM) had been frequently used for the gear mesh stiffness calculation [5]. In FEM, a very fine mesh is required to model the contact pattern in gears, which makes it computationally expensive. Wu et al. [6] proposed an analytical model for computing the TVMS of the crack tooth at different levels. They also observed the effect of tooth crack propagation on the vibration response of a one stage gearbox.

Till now, several analytical methods have been developed for the evaluation of time variable mesh stiffness of healthy as well as cracked gears [7–10]. Pandya et al. [11] developed an analytical model to observe the effect of crack nucleation and the rim thickness on the crack paths. They considered two gear parameters viz. backup ratio and pressure angle for high contact ratio spur gear with tooth root crack. They computed mesh stiffness variation using a total potential energy method. Chaari et al. [12] determined the reduction of mesh stiffness due to the presence of crack for tooth crack in spur gear. They incorporated tooth foundation deformation suggested by Sainsot et al. [13]. Chen and Shao [7, 14] proposed an analytical model to investigate the effect of gear tooth crack on the TVMS. They observed the effect of tooth crack propagation along the tooth width and along the crack depth on TVMS. Ghaffari et al. [15] performed 3-D finite element analysis to study the crack initiation and propagation behaviour in a gear tooth. They employed damage criterion to model the crack initiation on the surface of a gear tooth and observed the effect of friction on the fatigue crack initiation life. Ma et al. [16] investigated the effect of gear crack propagation on the dynamic behavior of a perforated gear system. They observed that the gear crack propagating through the rim has a larger effect on the vibration response than the gear crack propagating through the tooth under the same crack level. The model analysis of a geared rotor system is investigated using finite element method [17]. The authors performed a model analysis of a cracked gear rotor system to find out the effect of cracked gear tooth on the natural frequencies and the frequency response functions. An analytical method to calculate the TVMS of the spur gear for different spall shapes, size and location considering sliding friction is proposed [18]. They found the reduction in TVMS with the increase in spall length. In addition to this, they also observed that the unsymmetrical spall reduces the TVMS more than the symmetrical spall and circular, V-shaped spall creates a nonlinear reduction in TVMS. Ma et al. [19] proposed an analytical model for calculating TVMS of spur gears with the spalling defect. They concluded that the proposed analytical method is superior to FE method in terms of computational efficiency. Luo et al. [20] come up with a kinematic model to evaluate the TVMS of healthy and cracked gear teeth with constant center distance variation and time varying center distance variation. The obtained results indicate that the gear center distance variation has the significant effect on TVMS.

Till now, most of the work on gear tooth crack have been done using the analytical approach or finite element method. The analytical models show good agreement with the FE results and are computationally less expensive. Complex geometry and complex crack pattern problems are difficult to analyze by analytical methods. Although, FEM is a very powerful method and has been used extensively, it has some limitation due to very fine mesh requirement at the crack tip and the remeshing during the crack propagation. To overcome these limitations of FEM, an extended finite element method has been developed by Belytschko and Black [21]. In XFEM, Heaviside function is used as an additional enrichment function to model displacement discontinuity and the asymptotic enrichment function to model crack tip singularity [22]. The work on XFEM for the crack growth modeling in spur gear tooth is very limited. In this paper, XFEM has been extended to evaluate the time variable mesh stiffness in spur gear with different size of the crack. The crack is considered as of constant along tooth width and varies along the tooth thickness. Further, this method is extended to predict the crack growth behaviour in gear tooth for various rim thickness.

This paper is organized as follows: In the first section, the effectiveness of time-varying mesh stiffness estimation in gear condition monitoring is discussed. Importance of XFEM method over FEM method is also briefed in this section. An analytical model for the computation of time-varying mesh stiffness of spur gear pair with tooth root crack is thoroughly described in Section 2. Section 3 provides a description of XFEM for the computation of mesh stiffness and its comparison with the analytical results. In Section 4, a modified analytical method for mesh stiffness calculation is described. Further, crack propagation problem is discussed in section 5. The conclusions is outlined in Section 6.

2. Analytical model for computation of mesh stiffness in gear with tooth root crack

A tooth root crack is nucleated at the position in a gear tooth where stress concentration is developed during operating conditions [6]. The presence of tooth root crack causes a reduction in the effective tooth thickness. For the calculation of TVMS of spur gear with crack, tooth thickness is reduced due to the dead zone of tooth material. The dead zone of tooth material is defined as the area of tooth confined by tooth profile, tooth crack and the limit line [6, 14].

Gear mesh stiffness interprets the ability of the material to resist deformation when gears meshing. Gear mesh stiffness is a time-varying factor during the operation of gears. It depends on the number of engaged teeth, the applied load's position, direction, tooth geometry, gear's material specifications, profile errors and/or faults in the gear tooth [1, 12, 14]. In the present study, it is assumed that the crack is along the tooth thickness with a constant crack width. A gear mesh stiffness calculation model is proposed in Section 2.1.

2.1. Mesh stiffness calculation with crack through the tooth width

The stiffness of one tooth is a combination of the bending, shear, and the axial compressive stiffness, with all of them acting in the direction of the applied load. Deflection of a spur gear tooth under the action of the force can be determined by considering it as a non-uniform cantilever beam with an effective length d as shown in Fig. 1. The crack is supposed to go through the whole tooth width