Engineering Failure Analysis 84 (2018) 185–195

Contents lists available at ScienceDirect

Engineering Failure Analysis

journal homepage: www.elsevier.com/locate/engfailanal

Elasticity solution for the casing under linear crustal stress

Zhanfeng Chen[a,b], Weiping Zhu[a,*], Qinfeng Di[a,*]

[a] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, PR China.
[b] Institute of Process Equipment, College of Energy Engineering, Zhejiang University, Hangzhou 310027, PR China

A R T I C L E I N F O

Keywords:
Elasticity solution
Linear load
Casing
Stress function method

A B S T R A C T

The integrity of the casing is crucial for oil and gas well. Based on stress function method, a three-dimensional model of the casing-cement sheath-formation system subjected to linear crustal stress is proposed. And then an analytical solution of the model was obtained. In the process of calculation, the casing and cement sheath are simplified as the perfect cylinder. The cement sheath is closely bonded with the casing and formation. The formation is considered to be an isotropic material without the layer-block structure. And the crustal stress is assumed to be linearly increasing with the depth of the well. The analytical solution strictly meets the stress and displacement continuity condition and boundary condition, and exhibits good agreement with finite element method. The results imply that an analytical method to capture the stress and displacement field of the casing under linear crustal stress along the axis is presented. Next, a benchmark for numerical and approximate solutions is provided. In addition, a new idea about solving the casing under the non-linear loads along the axis in some special stratum (such as heterogeneity stratum, salt rock) is proposed. Finally, our understanding for the casing under complex loads will be deepened.

1. Introduction

In the process of oil exploration and development, casing is widely applied to protect well equipment and wellbore trajectory against the high crustal stress. Meanwhile, the cement sheath also provides structural support and zonal isolation through the entire service time of the well. Generally, even if the casing is located in the center of the well without wear or corrosion, the variations of the crustal stresses are possible to induce the failure of the casing and threaten the security of the oil and gas well. However, there are few scholars studied the effects of the linear crustal stress on the casing. Therefore, it is necessary to develop a convenient method to analyze the influence of crustal stresses on the casing.

The stress analysis of the casing under complex loads is a worldwide problem. The aim of this paper is to obtain the stresses of the casing under the growing linear crustal stress. Around this problem, Scholars have developed plenty of models to calculate casing stress distributions, and gained many theoretical and empirical equations. Early work on the stress distributions of the casing under external pressure can be traced back to Bryan [1] who obtained a limit stress equation for the long thin pipes. Southwell [2] derived another limit stress equation for the short pipes under external pressure. Unfortunately, his equation included an undetermined coefficient. The value of the coefficient was determined by Cook [3]. Von Mises [4] analyzed the collapse strength of short thin pipes subjected to lateral pressure without undetermined constants. Later, he extended his work to include both lateral and end loads [5]. Clinedinst [6] presented the ultimate bearing capacity of the casings subjected to external pressure. However, he ignored some important mechanical parameters such as the yield strength. Holmquist and Nadai [7] discussed the critical plastic pressure of the

* Corresponding authors.
E-mail addresses: wpzhu@shu.edu.cn (W. Zhu), qinfengd@sina.com (Q. Di).

https://doi.org/10.1016/j.engfailanal.2017.11.007
Received 20 June 2017; Received in revised form 8 September 2017; Accepted 13 November 2017
Available online 14 November 2017
1350-6307/

Nomenclature

a_0	internal radius of casing
a_1	external radius of casing (internal radius of cement sheath)
a_2	radius of well (external radius of cement sheath)
a_3	radius of formation
E_1, E_2, E_3	Young's modulus of the casing, cement sheath, and formation, respectively
G_1, G_2, G_3	shear modulus of the casing, cement sheath, and formation, respectively
h	height of the casing, cement sheath and formation
h_i	height of each section of the casing, cement sheath and formation
k_{s_1}	increment of s_1 from the top to the bottom along the axial direction
k_{s_2}	increment of s_2 from the top to the bottom along the axial direction
k_1	increment of p_1 from the top to the bottom along the axial direction
k_0	increment of p_0 from the top to the bottom along the axial direction
k_{p_i}	increment of p_i from the top to the bottom along the axial direction
k_{p_c}	increment of p_c from the top to the bottom along the axial direction
p_1	external pressure of a single casing on the top
$p_1^{'}$	external pressure of a single casing on the bottom
p_0	internal pressure of a single casing on the top
$p_0^{'}$	internal pressure of a single casing on the bottom
p_c	external pressure of the formation on the top
p'_c	external pressure of the formation on the bottom
p_i	internal pressure of the casing
q	initial axial stress of the pipe, cement sheath, and formation
r_1, r_2	internal and external radii of the pipe, respectively
s_1	interaction force between the casing and cement sheath at the top of the model
s_2	interaction force between the cement sheath and formation
${u_r}^1, {u_r}^2, {u_r}^3$	radial displacement of the casing, cement sheath, and formation, respectively
w^1, w^2, w^3	axial displacement of the casing, cement sheath, and formation, respectively
${\sigma_r}^1, {\sigma_\theta}^1, {\sigma_z}^1$	radial stress, hoop stress, and axial stress of the casing, respectively
${\sigma_r}^2, {\sigma_\theta}^2, {\sigma_z}^2$	radial stress, hoop stress, and axial stress of the cement sheath, respectively
${\sigma_r}^3, {\sigma_\theta}^3, {\sigma_z}^3$	radial stress, hoop stress, and axial stress of the formation, respectively
μ^1, μ^2, μ^3	Poisson's ratio of the casing, cement sheath, and formation, respectively

casing. Von Sanden and Gunther [8] studied the stress distributions of the stiffened pipes under the uniform external pressure. Timoshenko and Gere [9] conducted elastic stability analyses for a pipeline subjected to external pressure. Zhao [10] obtained an analytical solution of the oval casing under the external pressure. In recent years, Tamano et al. [11], Issa and Crawford [12], Tokimasa and Tanaka [13], and Huang et al. [14] have been engaged in studies of the effect of geometric imperfections (such as the eccentricity and ovality) on the stress and deformation of the casings, and have presented some effective empirical equations by using finite element methods. El-Sayed and Khalaf [15], Han and Shi [16,17], Yin et al. [18], Li et al. [19], and Yin and Gao [20] focused on the stress distribution of the casing subjected to non-uniform external pressure in the radial direction. Deng et al. [21] studied the high collapse casing (HCC) and its application in ultra-deep well. Huang and Gao [22] present a theoretical equation of the casing under the non-uniform loads in the radial direction. Yu et al. [23] researched the casing wear in a directional well under in situ stress. Non-uniform is common in nature, while the study on the stress distributions of the casing under non-uniform along the axis are lacking so far.

In this paper, we preliminarily discussed the stress distributions of the casing-cement sheath-formation system under the linear crustal stresses in theory. This is one kind of the casing under non-uniform along the axis. Our research is beneficial to provide a new method to analyze the stress distributions of the casing-cement sheath-formation system underground. In addition, our results can be applied to analyze the stresses and deformation of the casing in some special stratum (such as heterogeneity stratum, salt rock).

2. Model and basic hypotheses of the problem

As the crucial devices of oil and gas industry, the security of the casing captures an ocean of scholars and organizations' attention.

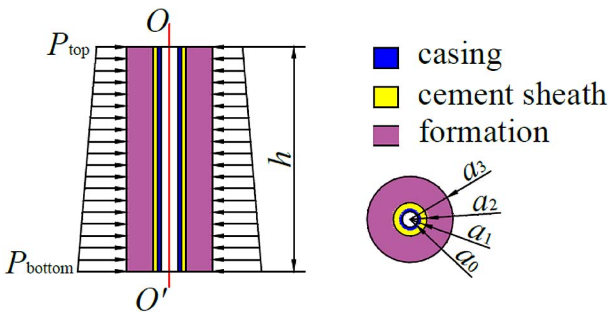

Fig. 1. Model of the casing-cement sheath-formation system. $OO^{'}$ is the axis of the model.