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Cyclic *J*-integral: Numerical and analytical investigations for surface cracks in weldments

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ABSTRACT

The cyclic *J*-integral (ΔJ-integral) is a crack tip parameter of elastic-plastic fracture mechanics which can be used as governing parameter for the description of fatigue crack growth (FCG) in metallic structures. In this contribution, it is applied for modelling FCG in weldments. The ΔJ-integral is determined by means of analytical approximation formulas as well as numerical methods. An analytical solution, which takes into account effects of the local ligament plasticity, was derived. This solution is based on well established methods such as R6, BS7910 and SINTAP which were modified for cyclic loading. It incorporates methods for the description of short crack closure behaviour as well as the well known analytical (long) crack closure function of Newman. A specific code was written to evaluate the ΔJ-integral numerically in the course of finite element based crack growth simulations. The code was first validated for an infinite plate with centre crack by applying elastic and elastic-plastic material behaviour. Next, the ΔJ-integral was calculated for cracks in various butt and cruciform welded joints. The results were compared with the results of the derived analytical approximation formula. A good accordance was achieved between the results. Note that the work was part of the German research cluster IBESS the aim of which was the development of a method for fracture mechanics based determination of the fatigue strength of weldments. Since the question behind the present paper was restricted to the cyclic elastic plastic crack driving force needed for the short fatigue crack propagation stage, only the geometrical aspects of weldments (i.e. the weld toe notch) are addressed here whilst other characteristics such as material inhomogeneity (HAZ) or residual stresses are discussed by other papers of this special issue.

1. Introduction

A correct description of FCG in a structural component can help to determine the functional life of the component and also to establish inspection intervals. Many studies [1–11] have shown that the lifetime estimation of welded joints can be done by using the fracture mechanics concept (crack propagation approach) only, because the crack nucleation stage is very short (due to the presence of defects in the weld), so that it can be neglected compared to crack propagation stages. The description of FCG in weldments is usually based on linear elastic fracture mechanics [11–15] where the small-scale

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Nomenclature

a	crack length (crack depth for surface cracks)
a/c	crack aspect ratio
c	half of the surface crack length (semi-elliptical crack)
C, m	coefficients of the crack growth equation, Eq. (1)
C_J, m_J	coefficients of the crack growth equation, Eq. (2)
C, m, p	coefficients of the crack growth equation, Eq. (32)
da/dN	increase in crack length per cycle
ds	infinitesimal arc length along the contour Γ for J-integral calculation
E, E'	Young's modulus, modified for plane strain
f	crack closure function, Eq. (33)
$f(\Delta L_r)$	plasticity corrected function, Eq. (30)
h	weld reinforcement, Figs. 6 and 7
J	(monotonic) J-integral
ΔJ	cyclic J-integral
K	stress intensity factor
K', n'	cyclic material parameters, Eq. (14)
ΔK_p	plasticity-corrected range of the K-factor, Eq. (31)
L	weld width, Figs. 6 and 7
L	load increment counter, Eq. (9)
L_{lower}, L_{upper}	lower and upper load reversal points, Eq. (9)
L_r	ligament yielding parameter for monotonic loading, Eq. (29)
ΔL_r	ligament yielding parameter for cyclic loading, Eq. (29)
N	number of load cycles, Eqs. (1) and (2)
n_j	local outward normal vector at the points of the contour Γ
R	nominal stress ratio, load ratio, K-factor ratio
S	nominal stress
δS	nominal stress increment, Fig. 12
t	time
T	plate thickness
T	nominal shear stress
T_i	component of the traction vector for determining the J-integral, Eq. (3)
u_i	component of the displacement vector for determining the J-integral, Eq. (3)
U, U^*	crack closure factors, Eqs. (32) and (35)
W	strain energy density
ΔW	cyclic counterpart of the strain energy density
x, y, z	Cartesian coordinates
α	weld flank angle, Figs. 6 and 7
α_g	constraint coefficient, Eqs. (24) and (25)
δx	auxiliary variable, Eqs. (11) and (12)
ε	strain
ε_{ij}	strain tensor components
Γ	path for J-integral determination, Eq. (3)
ν	Poisson's ratio
ρ	weld toe radius
σ	stress
σ_{app}	applied stress amplitude (referring to the gross section), Eq. (29)
σ_{ij}	stress tensor components
σ_0	flow stress, Eq. (24)
σ_0	reference yield stress, Eq. (29)
σ_y	yield stress, Eq. (15)

Indices

a	amplitude
app	applied
cl	closed (value at crack closure)
op	opened (value at crack opening)
max	maximum value in a cycle
min	minimum value in a cycle
e	elastic component