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Determination of crack tip stress intensity factors by singular Voronoi cell finite element model

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ABSTRACT

A singular Voronoi cell finite element model (SVCFEM) is proposed for estimation of the mixed-mode stress intensity factors (SIFs) of crack tip in this paper. Formulation of a singular Voronoi cell finite element is based on a modified complementary energy principle. To satisfy the stress singularity at the crack tip, we enrich stress solution in assumed stress hybrid model. In addition to polynomial and reciprocal terms, singular stress terms of Williams expansion are added for elliptical crack to capture crack-tip stress concentrations. After obtaining the stress, SIFs of model I and model II were calculated using linear least-squares method. Comparisons of SVCFEM solution with analytical solution for crack are made to demonstrate the efficiency of SVCFEM.

1. Introduction

The ability to predict brittle fracture of engineering accessory and structures in manufacturing and service are very important. Stress intensity factors are important for fracture analysis of cracked structures, because it have been used for the prediction of crack initiation, crack propagation direction and speed in brittle materials.

In recent decades, various methods including experimental test, numerical analysis and simulation have been developed, from which finite element calculation can provide more convenient means for optimal design. When conventional finite element method is applied to calculate the fracture problem, many high-density grids are required. Because the strain and stress calculation of crack tip does not include singularity. In this case, no matter how high order polynomial is used for the assumed displacement function, the convergence rate is very slow due to very small element around crack tip. To improve the calculation efficiency, many improved finite element methods are proposed through better representing crack tip singularity. The singular element [1,2] has been proposed by placing the mid-side node front at the quarter-point position or near the crack tip to yield reasonable SIFs and displacement. These singular elements reproduce square root stress singularity. Tong et al. [3] proposed a hybrid crack-tip element using classical elasticity theory. Dan Zeng [4] develop hybrid crack-tip element for crack surface with traction. The hybrid element also named super-element apply conformal mapping technique to build stress functions and displacement fields. For simplicity, direct approximation of the stress and boundary displacement fields are applied in hybrid-Trefftz stress element [5].

Moorthy and Ghosh [6] proposed Voronoi cell finite element to describe the micromechanical properties of composite materials. The location distribution, shape and size of the heterogeneity like an inclusion or crack are arbitrary as can be seen from real microscopic picture. Tessellate the real microstructure into a mesh of Voronoi polygons, and each Voronoi cell contain one heterogeneity at most. The stress and deformation of each point in the matrix are under the influence of these complex topology structure, and the greatest influence is suffered from the inclusion closest to this point, so all points closest to inclusion constitute a geometry body and combined as an element. Every element side is formed by the center bisector of interior inclusion and the adjacent

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inclusion. In this paper, every element contains a preset crack and the surrounding matrix. So VCFEM provide a direct relationship between computation geometry and deformation analysis and can be used effectively. The computational efficiency is greatly improved due to significantly reduced degrees of freedom compared to conventional finite element models has been used to analyze the interfacial degrading problem of particle reinforced composites. VCFEM [7,8] has been used to analyze the interfacial debonding problem of particle or fiber reinforced composites.

Zhang [9] proposed parametric variational principle based polygonal finite element method (PFEM) and VCFEM for simulations of heterogeneous materials. Zhang [10] proposed a modified VCFEM which account for the presence of fluid pressure within porous materials. In the above literatures, the assumed stress function is composed with a purely polynomial function and a reciprocal function. The selected stress function is suitable for inclusion of ellipses. However, for crack problem, that selection of stress function will failed to capture stress concentration near crack tip. To solve such problem, Li and Ghosh [11] introduced wavelet function to construct the stress function and obtain reasonable stress result, also SIFs were calculated by the J-integral method in their research. However, wavelet parameters such as the translation parameter and the dilation parameter need to be chosen carefully to avoid false stress, and the level for different stress field is complicated to choose to use strain energy error method in the literature.

The purpose of this paper is to proposed a new alternate approach for evaluating stress concentration around crack tip in VCFEM which gives effective results without cumbersome parameter selection, and SIFs are calculated from post-processing. The construction of stress function has important influence on the convergence of VCFEM, so it is a key step in Voronoi cell element formulation. The heterogeneity in an element is crack in this paper, so we take consideration on stress concentration near crack tip or singularity at crack tip, and singular stress terms of Williams are superimposed on stress fields directly. We named this method as singular Voronoi cell element method(SVCFEM). Subsequently, a linear least square method is used to calculate SIFs.

2. SVCFEM formulation for crack

Fig. 1(a) shows a pre-crack microstructure containing random distribution cracks with each crack tesselate into a polygon Voronoi cell element. A typical Voronoi cell element containing a crack and its surrounding matrix is depicted in Fig. 1(b). The element boundary $\partial\Omega_e$ with an outward normal $\mathbf{n}^e$ is composed of prescribed displacement boundary Γ_{um}, prescribed traction boundary Γ_{tm}, inter-element boundary Γ_m and free boundary $\partial\Omega_e^f$, i.e., $\partial\Omega_e = \Gamma_{um} \cup \Gamma_{tm} \cup \Gamma_m \cup \Gamma_{mf}$. According to the minimum complimentary energy principle, the energy function is generally written as

$$\Pi_c = -\int_{\Omega_e} \frac{1}{2}\boldsymbol{\sigma}: \mathbf{S}: \boldsymbol{\sigma} d\Omega + \int_{\Gamma_{um}} \mathbf{T}\cdot\bar{\mathbf{u}} d\partial\Omega \tag{1}$$

where, $\mathbf{S}$ is the material compliance matrix. To satisfy the prescribed traction boundary conditions. The following condition are satisfied as $\boldsymbol{\sigma}\cdot\mathbf{n} = \overline{\mathbf{T}}$ on Γ_{tm} and $\boldsymbol{\sigma}\cdot\mathbf{n}^+ = \boldsymbol{\sigma}\cdot\mathbf{n}^-$ on Γ_m. The above constraint conditions are implemented by the Lagrange multiplier method. Then energy functional is written as

$$\Pi_{mc} = -\int_{\Omega_e} \frac{1}{2}\boldsymbol{\sigma}: \mathbf{S}: \boldsymbol{\sigma} d\Omega + \int_{\Gamma_{um}} \mathbf{T}\cdot\bar{\mathbf{u}} d\partial\Omega + \int_{\Gamma_m} \boldsymbol{\sigma}\cdot\mathbf{n}^e\cdot\mathbf{u}^e d\partial\Omega + \int_{\Gamma_{tm}} (\boldsymbol{\sigma}\cdot\mathbf{n}^e - \overline{\mathbf{T}})\cdot\mathbf{u}^e d\partial\Omega \tag{2}$$

where, the boundary force is $\mathbf{T} = \boldsymbol{\sigma}\cdot\mathbf{n}$; Given displacement also satisfies the element displacement interpolation, so $\bar{\mathbf{u}} = \mathbf{u}$. In this way, energy function can be written as

$$\Pi_{mc} = \sum_e \left(-\int_{\Omega_e} \frac{1}{2}\boldsymbol{\sigma}: \mathbf{S}: \boldsymbol{\sigma} d\Omega + \int_{\partial\Omega_e} \boldsymbol{\sigma}\cdot\mathbf{n}^e\cdot\mathbf{u}^e d\partial\Omega - \int_{\Gamma_{tm}} \overline{\mathbf{T}}\cdot\mathbf{u}^e d\partial\Omega \right) \tag{3}$$

In this paper, pre-existing elliptical shape cracks Γ_{cr} with a normal $\mathbf{n}^{cr}$ is assumed to be elongated elliptic. The modified energy functional is as follows:

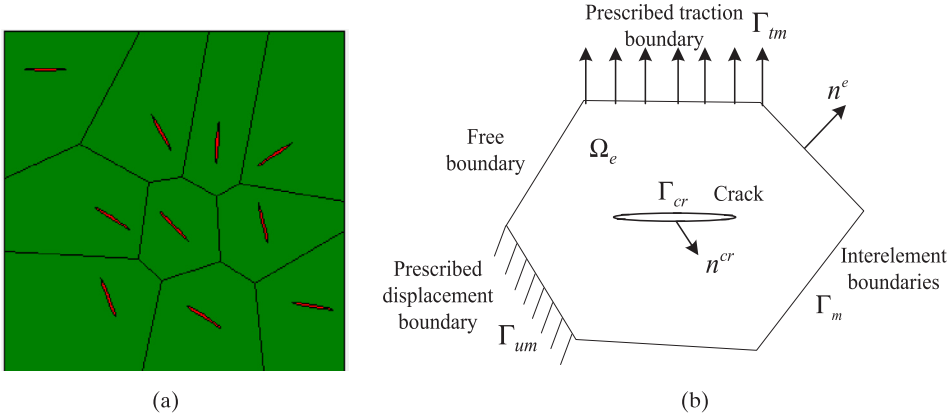

Fig. 1. (a) A mesh of Voronoi cell elements, (b) a typical Voronoi cell element showing boundary features.