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# A simple model to explain transferability of crack tip opening angle

#### Longkun Lu, Shengnan Wang\*

School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, China

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#### ABSTRACT

A stable crack growth model based on critical crack tip opening angle (CTOA) is proposed. This model considers the growing crack opening profile as corresponding stationary crack opening profile minus plastic wake profile. A relationship between CTOA and stationary crack tip opening displacement is established. Relations between CTOA and *K*-R curves, *J* integral are then obtained from this relationship and the crack growth model is thus verified. Based on this model, the transferability of CTOA is discussed by dimensional analysis. The critical CTOA is independent of initial crack size, structural size, and structural configuration for a given material and stress state if the plastic zone stress distribution is invariable. A method to predict residual strength based on a critical CTOA is proposed and rudimentary verification is performed using previously published fracture test results.

#### 1. Introduction

As cracks propagate stably in elastic plastic materials, the yielding region is not small and plastic unloading will occur, and then the linear fracture mechanics and elastic plastic fracture parameters based on deformation theory of plasticity will cease to be valid [1]. Crack tip opening angle (CTOA) or displacement (CTOD) at a specified distance from crack tip seems to be most suitable for modeling the stable crack growth case [2].

CTOA reflects the local slope of crack faces near the crack tip and has been found to be nearly constant during stable crack growth [2–4]. In addition, CTOA was found to be independent of in-plane geometry if the crack length and un-cracked ligament size are larger than 4 times thickness [5]. Thus, CTOA was widely used to predict stable crack growth of thinned wall structures, such as stiffened and stiffened with or without multiple sites damage [6–9] during the past three decades. ASTM has also established a standard procedure to measure critical CTOA from C (T) or M (T) specimens [10]. However, a theoretical analysis is needed to explain the constant nature of CTOA when crack length and un-cracked ligament size are large enough. Existing literature explanations [5] are based on incremental finite element analysis and only applied to elastic perfectly plastic materials. Thus, a theoretical analysis to discuss transferability of CTOA is required. Meanwhile, the critical CTOA concept has been used to predict stable crack growth computationally by intensive finite element techniques [2–9]. A simple but reliable engineering method is required to allow the CTOA based fracture criterion applicable to common engineering fracture control issues.

A crack tip opening displacement  $V_R$  was defined by Newman [11,12] as the corresponding stationary crack tip opening displacement of a growing crack, and the  $V_R - \Delta a$  curve was found to be independent of crack length, specimen width and

\* Corresponding author. E-mail address: wangshna@nwpu.edu.cn (S. Wang).

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x, $x_t$ a coordinate centered at the crack tip and with the opposite direction of $x_1$ $(x_1, x_2)$ cartesian coordinates $\alpha$ material constant $\beta$ constraint factor $\Delta a$ crack extension, mm $\delta_0$ critical initiation crack opening displacement, mm $\delta_a$ the advancing crack opening profile, mm $\delta_a$ crack drive force in terms of crack tip opening profile, mm $\delta_{\alpha}$ the plastic wake profile, mm $\delta_{\alpha}$ resistance to crack extension in terms of crack tip opening profile, mm $\delta_{k}$ resistance to crack extension in terms of crack tip opening profile, mm $\delta_{k}^{c}$ $\delta_{R} - \Delta a$ curves of M (T) and C (T) specimens respectively, mm $\delta_{k}^{c}$ $\delta_{R} - \Delta a$ curves of infinite panels $\delta_{t}$ the stationary crack opening profile, mm $\theta^{o}$ a length similar to the plastic zone size under small scale yielding condition $\sigma$ applied load, MPa $\sigma_{0}$ flow stress, ( $\sigma_{ys} + \sigma_{u}$ )/2, MPa $\sigma_{u}$ ultimate tensile strength, MPa $\Theta_{a}$ a generalized dimensionless damage integral $\Theta_{c}$ critical crack tip opening angle, degreesCTOAcrack tip opening angleCTOAcrack tip opening angle	Ŵ	specimen width, mm	
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	$x, x_t$	a coordinate centered at the crack tip and with the opposite direction of $x_1$	
$\begin{array}{lll} \begin{array}{lll} \hline \alpha & \mbox{material constant} \\ \hline \beta & \mbox{constraint factor} \\ \hline \Delta a & \mbox{crack extension, mm} \\ \hline \Delta a & \mbox{crack extension, mm} \\ \hline \Delta a & \mbox{crack extension, mm} \\ \hline \delta_a & \mbox{the advancing crack opening profile, mm} \\ \hline \delta_a & \mbox{the advancing crack opening profile, mm} \\ \hline \delta_a & \mbox{crack drive force in terms of crack tip opening profile, mm} \\ \hline \delta_a & \mbox{crack drive force in terms of crack tip opening profile, mm} \\ \hline \delta_a & \mbox{crack drive force in terms of crack tip opening profile, mm} \\ \hline \delta_a & \mbox{crack drive force in terms of crack tip opening profile, mm} \\ \hline \delta_a & \mbox{crack extension in terms of crack tip opening profile, mm} \\ \hline \delta_B & \mbox{resistance to crack extension in terms of crack tip opening profile, mm} \\ \hline \delta_R^{K} & \mbox{\delta}_R^{K} & \mbox{\delta}_R - \Delta a \mbox{curves of M} (T) \mbox{and C} (T) \mbox{specimens respectively, mm} \\ \hline \delta_R^{K} & \mbox{\delta}_R^{K} - \Delta a \mbox{curves of infinite panels} \\ \hline \delta_i & \mbox{the stationary crack opening profile, mm} \\ \hline \delta^P & \mbox{plastic strain} \\ \hline \rho & \mbox{a length similar to the plastic zone size under small scale yielding condition} \\ \hline \sigma & \mbox{applied load, MPa} \\ \hline \sigma_u & \mbox{ultimate tensile strength, MPa} \\ \hline \sigma_{ys} & \mbox{yield stress} (0.2\% \mbox{ offset}), MPa \\ \hline \Theta & \mbox{a generalized dimensionless damage integral} \\ \hline \Theta_c & \mbox{critical crack tip opening angle, degrees} \\ \hline CTOD & \mbox{crack tip opening angle} \\ \hline CTOD & \mbox{crack tip opening displacement} \\ \hline \end{array}$	$(x_1, x_2)$	cartesian coordinates	
βconstraint factorΔacrack extension, mm $\delta_0$ critical initiation crack opening displacement, mm $\delta_a$ the advancing crack opening profile, mm $\delta_a$ crack drive force in terms of crack tip opening profile, mm $\delta_c$ critical crack opening displacement, mm $\delta_t$ the plastic wake profile, mm $\delta_{R}$ $\epsilon_{R}$ – Δa curves of M (T) and C (T) specimens respectively, mm $\delta_{R}^{e}$ $\delta_{R} - \Delta a$ curves of infinite panels $\delta_t$ the stationary crack opening profile, mm $\delta_{R}^{e}$ $\delta_{R} - \Delta a$ curves of infinite panels $\delta_t$ the stationary crack opening profile, mm $\rho^{P}$ plastic strain $\rho$ a length similar to the plastic zone size under small scale yielding condition $\sigma$ applied load, MPa $\sigma_u$ ultimate tensile strength, MPa $\sigma_{ys}$ yield stress (0.2% offset), MPa $\Theta$ a generalized dimensionless damage integral $\Theta_c$ critical value of $\Theta$ $\psi_c$ critical value of $\Theta$ $\psi_c$ critical crack tip opening angle, degreesCTODcrack tip opening angle	α	material constant	
$\begin{array}{lll} \Delta a & {\rm crack\ extension\ mm} \\ \delta_0 & {\rm critical\ initiation\ crack\ opening\ displacement\ mm} \\ \delta_a & {\rm the\ advancing\ crack\ opening\ profile\ mm} \\ \delta_a & {\rm track\ drive\ force\ in\ terms\ of\ crack\ tip\ opening\ profile\ mm} \\ \delta_A & {\rm crack\ drive\ force\ in\ terms\ of\ crack\ tip\ opening\ profile\ mm} \\ \delta_c & {\rm critical\ crack\ opening\ displacement\ mm} \\ \delta_H & {\rm th\ plastic\ wake\ profile\ mm} \\ \delta_H & {\rm th\ plastic\ wake\ profile\ mm} \\ \delta_R & {\rm resistance\ to\ crack\ extension\ in\ terms\ of\ crack\ tip\ opening\ profile\ mm} \\ \delta_R & \delta_R & - \Delta a\ curves\ of\ M\ (T)\ and\ C\ (T)\ specimens\ respectively\ mm} \\ \delta_R & \delta_R & - \Delta a\ curves\ of\ M\ (T)\ and\ C\ (T)\ specimes\ respectively\ mm} \\ \delta_R & \delta_R & - \Delta a\ curves\ of\ inite\ panels \\ \delta_T & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & \delta_R & - \Delta a\ curves\ of\ inite\ panels \\ \delta_T & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & \delta_R & - \Delta a\ curves\ of\ inite\ panels \\ \delta_T & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & \delta_R & - \Delta a\ curves\ of\ inite\ panels \\ \delta_T & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & \delta_R & - \Delta a\ curves\ of\ inite\ panels \\ \delta_T & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & \delta_R & \delta_R & - \Delta a\ curves\ of\ inite\ panels \\ \delta_T & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & \delta_R & \delta_R & - \Delta a\ curves\ of\ inite\ panels \\ \delta_T & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & \delta_R & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & {\rm th\ estationary\ crack\ opening\ profile\ mm} \\ \delta_R & {\rm th\ estatic\ strain} \ \delta_R & {\rm th$	β	constraint factor	
$\begin{array}{lll} \delta_0 & {\rm critical\ initiation\ crack\ opening\ displacement,\ mm} \\ \delta_a & {\rm the\ advancing\ crack\ opening\ profile,\ mm} \\ \delta_A & {\rm crack\ drive\ force\ in\ terms\ of\ crack\ tip\ opening\ profile,\ mm} \\ \delta_A & {\rm crack\ drive\ force\ in\ terms\ of\ crack\ tip\ opening\ profile,\ mm} \\ \delta_c & {\rm critical\ crack\ opening\ displacement,\ mm} \\ \delta_H & {\rm th\ e\ plastic\ wake\ profile,\ mm} \\ \delta_R & {\rm resistance\ to\ crack\ extension\ in\ terms\ of\ crack\ tip\ opening\ profile,\ mm} \\ \delta_R & {\rm crack\ drive\ force\ in\ terms\ of\ crack\ tip\ opening\ profile,\ mm} \\ \delta_R & {\rm crack\ extension\ in\ terms\ of\ crack\ tip\ opening\ profile,\ mm} \\ \delta_R & {\rm crack\ extension\ in\ terms\ of\ crack\ tip\ opening\ profile,\ mm} \\ \delta_R^{\infty} & \delta_R - \Delta a\ curves\ of\ M\ (T)\ and\ C\ (T)\ specimens\ respectively,\ mm \\ \delta_R^{\infty} & \delta_R - \Delta a\ curves\ of\ infinite\ panels \\ \delta_R & {\rm th\ e\ stationary\ crack\ opening\ profile,\ mm} \\ \delta_R^{\sigma} & \delta_R - \Delta a\ curves\ of\ infinite\ panels \\ \delta_R & {\rm th\ e\ stationary\ crack\ opening\ profile,\ mm} \\ \delta_R^{\sigma} & \delta_R - \Delta a\ curves\ of\ infinite\ panels \\ \delta_R & {\rm th\ e\ stationary\ crack\ opening\ profile,\ mm} \\ \delta_R^{\sigma} & \delta_R - \Delta a\ curves\ of\ infinite\ panels \\ \delta_R & {\rm th\ e\ stationary\ crack\ opening\ profile,\ mm} \\ \delta_R^{\sigma} & \delta_R - \Delta a\ curves\ of\ infinite\ panels \\ \delta_R & {\rm th\ e\ stationary\ crack\ opening\ profile,\ mm} \\ \delta_R^{\sigma} & {\rm a\ length\ similar\ to\ th\ plastic\ zone\ size\ under\ small\ scale\ yielding\ condition \\ \sigma & {\rm a\ plastic\ strass\ } (\sigma_{ys} + \sigma_u)/2,\ MPa \\ \sigma_u & {\rm ultimat\ te\ ssile\ strass\ damage\ integrall \\ \Theta_{\sigma} & {\rm a\ generalized\ dimensionless\ damage\ integrall} \\ \Theta_{\sigma} & {\rm a\ generalized\ dimensionless\ damage\ integrall \\ \Theta_{\sigma} & {\rm critical\ value\ of\ \Theta} \\ \psi_c & {\rm critical\ value\ of\ \Theta} \\ \psi_c & {\rm critical\ value\ of\ \Theta} \\ \psi_c & {\rm critical\ value\ do\ \Theta} \ \ \psi_c &$	Δa	crack extension, mm	
$\begin{array}{ll} \delta_a & \text{the advancing crack opening profile, mm} \\ \delta_A & \text{crack drive force in terms of crack tip opening profile, mm} \\ \delta_c & \text{critical crack opening displacement, mm} \\ \delta_H & \text{the plastic wake profile, mm} \\ \delta_R & \text{the plastic wake profile, mm} \\ \delta_R & \text{tesistance to crack extension in terms of crack tip opening profile, mm} \\ \delta_R^{\sigma} & \delta_R - \Delta a \text{ curves of M (T) and C (T) specimens respectively, mm} \\ \delta_R^{\sigma} & \delta_R - \Delta a \text{ curves of infinite panels} \\ \delta_R & \text{the stationary crack opening profile, mm} \\ \delta_R^{P} & \text{plastic strain} \\ \rho & \text{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \text{applied load, MPa} \\ \sigma_u & \text{ultimate tensile strength, MPa} \\ \sigma_y & \text{stress distributions over the plastic wake region} \\ \sigma_0 & \text{flow stress, } (\sigma_{ys} + \sigma_u)/2, \text{ MPa} \\ \Theta & \text{a generalized dimensionless damage integral} \\ \Theta_c & \text{critical value of } \Theta \\ \psi_c & \text{critical crack tip opening angle, degrees} \\ \text{CTOD} & \text{crack tip opening displacement} \end{array}$	$\delta_0$	critical initiation crack opening displacement, mm	
$\begin{array}{lll} \delta_A & {\rm crack \ drive \ force \ in \ terms \ of \ crack \ tip \ opening \ profile, \ mm} \\ \delta_c & {\rm critical \ crack \ opening \ displacement, \ mm} \\ \delta_H & {\rm the \ plastic \ wake \ profile, \ mm} \\ \delta_H & {\rm the \ plastic \ wake \ profile, \ mm} \\ \delta_R & {\rm resistance \ to \ crack \ extension \ in \ terms \ of \ crack \ tip \ opening \ profile, \ mm} \\ \delta_R & \delta_R & - \Delta a \ curves \ of \ M \ (T) \ and \ C \ (T) \ specimens \ respectively, \ mm \\ \delta_R^{m}, \delta_R^{r} & \delta_R & - \Delta a \ curves \ of \ M \ (T) \ and \ C \ (T) \ specimens \ respectively, \ mm \\ \delta_R^{m}, \delta_R^{r} & \delta_R & - \Delta a \ curves \ of \ M \ (T) \ and \ C \ (T) \ specimens \ respectively, \ mm \\ \delta_R^{m}, \delta_R^{r} & \delta_R & - \Delta a \ curves \ of \ mn \ (T) \ and \ C \ (T) \ specimens \ respectively, \ mm \\ \delta_R^{r} & \delta_R & - \Delta a \ curves \ of \ mn \ (T) \ and \ C \ (T) \ specimens \ respectively, \ mm \\ \delta_R^{r} & \delta_R^{r} & - \Delta a \ curves \ of \ mn \ (T) \ and \ C \ (T) \ specimens \ respectively, \ mm \\ \delta_R^{r} & \delta_R^{r} & - \Delta a \ curves \ of \ mn \ (T) \ and \ (T) \ (T$	$\delta_a$	the advancing crack opening profile, mm	
$\begin{array}{lll} \delta_c & \mbox{critical crack opening displacement, mm} \\ \delta_H & \mbox{the plastic wake profile, mm} \\ \delta_R & \mbox{critical crack extension in terms of crack tip opening profile, mm} \\ \delta_R & \delta_R^{-} & \Delta a \mbox{curves of M (T) and C (T) specimens respectively, mm} \\ \delta_R^{-} & \delta_R - \Delta a \mbox{curves of infinite panels} \\ \delta_R & \delta_R - \Delta a \mbox{curves of infinite panels} \\ \delta_t & \mbox{the stationary crack opening profile, mm} \\ \delta_P^{-} & \mbox{plastic strain} \\ \rho & \mbox{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \mbox{applied load, MPa} \\ \sigma(x) & \mbox{stress distributions over the plastic wake region} \\ \sigma_0 & \mbox{flow stress, } (\sigma_{ys} + \sigma_u)/2, \mbox{MPa} \\ \sigma_y & \mbox{yield stress} (0.2\% \mbox{offset}), \mbox{MPa} \\ \Theta & \mbox{a generalized dimensionless damage integral} \\ \Theta_c & \mbox{critical crack tip opening angle, degrees} \\ \mbox{CTOA} & \mbox{crack tip opening angle} \\ \mbox{CTOD} & \mbox{crack tip opening displacement} \end{array}$	$\delta_A$	crack drive force in terms of crack tip opening profile, mm	
$\begin{array}{ll} \delta_{H} & \mbox{the plastic wake profile, mm} \\ \delta_{R} & \mbox{resistance to crack extension in terms of crack tip opening profile, mm} \\ \delta_{R}^{M}, \delta_{C}^{R} & \delta_{R} - \Delta a \mbox{curves of M (T) and C (T) specimens respectively, mm} \\ \delta_{R}^{\infty} & \delta_{R} - \Delta a \mbox{curves of infinite panels} \\ \delta_{L} & \mbox{the stationary crack opening profile, mm} \\ \delta_{P}^{P} & \mbox{plastic strain} \\ \rho & \mbox{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \mbox{applied load, MPa} \\ \sigma(x) & \mbox{stress distributions over the plastic wake region} \\ \sigma_{0} & \mbox{flow stress, } (\sigma_{ys} + \sigma_{u})/2, \mbox{MPa} \\ \sigma_{u} & \mbox{ultimate tensile strength, MPa} \\ \sigma_{ys} & \mbox{yield stress (0.2% offset), \mbox{MPa} \\ \Theta & \mbox{a generalized dimensionless damage integral} \\ \Theta_{c} & \mbox{critical crack tip opening angle, \mbox{degrees}} \\ \end{tabular}$	$\delta_c$	critical crack opening displacement, mm	
$\begin{array}{ll} \delta_{R} & \text{resistance to crack extension in terms of crack tip opening profile, mm} \\ \delta_{R}^{M}, \delta_{C}^{R} & \delta_{R} - \Delta a \text{ curves of M (T) and C (T) specimens respectively, mm} \\ \delta_{R}^{\infty} & \delta_{R} - \Delta a \text{ curves of infinite panels} \\ \delta_{t} & \text{the stationary crack opening profile, mm} \\ \varepsilon^{P} & \text{plastic strain} \\ \rho & \text{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \text{applied load, MPa} \\ \sigma(x) & \text{stress distributions over the plastic wake region} \\ \sigma_{0} & \text{flow stress, } (\sigma_{ys} + \sigma_{u})/2, \text{ MPa} \\ \sigma_{ys} & \text{yield stress (0.2% offset), MPa} \\ \Theta & \text{a generalized dimensionless damage integral} \\ \Theta_{c} & \text{critical crack tip opening angle, degrees} \\ \text{CTOA} & \text{crack tip opening displacement} \end{array}$	$\delta_H$	the plastic wake profile, mm	
$\begin{split} \delta^{A}_{R}, \delta^{C}_{L} & \delta_{R} - \Delta a \text{ curves of M (T) and C (T) specimens respectively, mm} \\ \delta^{\infty}_{R} & \delta_{R} - \Delta a \text{ curves of infinite panels} \\ \delta_{t} & \text{the stationary crack opening profile, mm} \\ \epsilon^{p} & \text{plastic strain} \\ \rho & \text{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \text{applied load, MPa} \\ \sigma(x) & \text{stress distributions over the plastic wake region} \\ \sigma_{0} & \text{flow stress, } (\sigma_{ys} + \sigma_{u})/2, \text{MPa} \\ \sigma_{u} & \text{ultimate tensile strength, MPa} \\ \sigma_{ys} & \text{yield stress (0.2\% offset), MPa} \\ \Theta & \text{a generalized dimensionless damage integral} \\ \Theta_{c} & \text{critical value of } \Theta \\ \psi_{c} & \text{critical crack tip opening angle, degrees} \\ \text{CTOD} & \text{crack tip opening displacement} \end{split}$	$\delta_R$	resistance to crack extension in terms of crack tip opening profile, mm	
$\begin{array}{ll} \delta_{R}^{\infty} & \delta_{R} - \Delta a \mbox{ curves of infinite panels} \\ \delta_{t} & \mbox{the stationary crack opening profile, mm} \\ \varepsilon^{p} & \mbox{plastic strain} \\ \rho & \mbox{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \mbox{applied load, MPa} \\ \sigma(x) & \mbox{stress distributions over the plastic wake region} \\ \sigma_{0} & \mbox{flow stress, } (\sigma_{ys} + \sigma_{u})/2, \mbox{MPa} \\ \sigma_{u} & \mbox{ultimate tensile strength, MPa} \\ \sigma_{ys} & \mbox{yield stress (0.2\% offset), MPa} \\ \Theta & \mbox{a generalized dimensionless damage integral} \\ \Theta_{c} & \mbox{critical crack tip opening angle, degrees} \\ CTOA & \mbox{crack tip opening displacement} \end{array}$	$\delta_R^M, \delta_R^C$	$\delta_R - \Delta a$ curves of M (T) and C (T) specimens respectively, mm	
$\begin{array}{ll} \delta_t & \text{the stationary crack opening profile, mm} \\ \varepsilon^p & \text{plastic strain} \\ \rho & \text{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \text{applied load, MPa} \\ \sigma(x) & \text{stress distributions over the plastic wake region} \\ \sigma_0 & \text{flow stress, } (\sigma_{ys} + \sigma_u)/2, \text{MPa} \\ \sigma_u & \text{ultimate tensile strength, MPa} \\ \sigma_{ys} & \text{yield stress (0.2\% offset), MPa} \\ \Theta & \text{a generalized dimensionless damage integral} \\ \Theta_c & \text{critical value of } \Theta \\ \psi_c & \text{critical crack tip opening angle, degrees} \\ \text{CTOD} & \text{crack tip opening displacement} \end{array}$	$\delta^{\infty}_R$	$\delta_R - \Delta a$ curves of infinite panels	
$\begin{array}{ll} \varepsilon^{p} & \text{plastic strain} \\ \rho & \text{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \text{applied load, MPa} \\ \sigma(x) & \text{stress distributions over the plastic wake region} \\ \sigma_{0} & \text{flow stress, } (\sigma_{ys} + \sigma_{u})/2, \text{MPa} \\ \sigma_{u} & \text{ultimate tensile strength, MPa} \\ \sigma_{ys} & \text{yield stress (0.2\% offset), MPa} \\ \Theta & \text{a generalized dimensionless damage integral} \\ \Theta_{c} & \text{critical value of } \Theta \\ \psi_{c} & \text{critical crack tip opening angle, degrees} \\ \text{CTOA} & \text{crack tip opening displacement} \end{array}$	$\delta_t$	the stationary crack opening profile, mm	
$\begin{array}{ll} \rho & \text{a length similar to the plastic zone size under small scale yielding condition} \\ \sigma & \text{applied load, MPa} \\ \sigma(x) & \text{stress distributions over the plastic wake region} \\ \sigma_0 & \text{flow stress, } (\sigma_{ys} + \sigma_u)/2, \text{MPa} \\ \sigma_u & \text{ultimate tensile strength, MPa} \\ \sigma_{ys} & \text{yield stress (0.2\% offset), MPa} \\ \Theta & \text{a generalized dimensionless damage integral} \\ \Theta_c & \text{critical value of } \Theta \\ \psi_c & \text{critical crack tip opening angle, degrees} \\ \text{CTOA} & \text{crack tip opening displacement} \end{array}$	$\varepsilon^p$	plastic strain	
$\sigma$ applied load, MPa $\sigma(x)$ stress distributions over the plastic wake region $\sigma_0$ flow stress, $(\sigma_{ys} + \sigma_u)/2$ , MPa $\sigma_u$ ultimate tensile strength, MPa $\sigma_{ys}$ yield stress (0.2% offset), MPa $\Theta$ a generalized dimensionless damage integral $\Theta_c$ critical value of $\Theta$ $\psi_c$ critical crack tip opening angle, degreesCTOAcrack tip opening displacement	ho	a length similar to the plastic zone size under small scale yielding condition	
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$\sigma_0$ flow stress, $(\sigma_{ys} + \sigma_u)/2$ , MPa $\sigma_u$ ultimate tensile strength, MPa $\sigma_{ys}$ yield stress (0.2% offset), MPa $\Theta$ a generalized dimensionless damage integral $\Theta_c$ critical value of $\Theta$ $\psi_c$ critical crack tip opening angle, degreesCTOAcrack tip opening angleCTODcrack tip opening displacement	$\sigma(x)$	stress distributions over the plastic wake region	
$\sigma_u$ ultimate tensile strength, MPa $\sigma_{ys}$ yield stress (0.2% offset), MPa $\Theta$ a generalized dimensionless damage integral $\Theta_c$ critical value of $\Theta$ $\psi_c$ critical crack tip opening angle, degreesCTOAcrack tip opening angleCTODcrack tip opening displacement	$\sigma_0$	flow stress, $(\sigma_{ys} + \sigma_u)/2$ , MPa	
$\sigma_{ys}$ yield stress (0.2% offset), MPa $\Theta$ a generalized dimensionless damage integral $\Theta_c$ critical value of $\Theta$ $\psi_c$ critical crack tip opening angle, degreesCTOAcrack tip opening angleCTODcrack tip opening displacement	$\sigma_u$	ultimate tensile strength, MPa	
$\Theta$ a generalized dimensionless damage integral $\Theta_c$ critical value of $\Theta$ $\psi_c$ critical crack tip opening angle, degreesCTOAcrack tip opening angleCTODcrack tip opening displacement	$\sigma_{ys}$	yield stress (0.2% offset), MPa	
$\begin{array}{lll} \Theta_c & \text{critical value of } \Theta \\ \psi_c & \text{critical crack tip opening angle, degrees} \\ \text{CTOA} & \text{crack tip opening angle} \\ \text{CTOD} & \text{crack tip opening displacement} \end{array}$	Θ	a generalized dimensionless damage integral	
$\psi_c$ critical crack tip opening angle, degrees CTOA crack tip opening angle CTOD crack tip opening displacement	$\Theta_c$	critical value of $\Theta$	
CTOD crack tip opening displacement	$\psi_c$	critical crack tip opening angle, degrees	
CTOD crack tip opening displacement	CTUA	crack tip opening angle	
	CIOD	crack tip opening displacement	

specimen type. Newman's work and the papers of Deng, Hutchinson and Nilsson et al. [13-16] have generated the idea that the growing crack profile is the superposition of the corresponding stationary crack opening profile and plastic wake profile, providing a direction to study CTOA's properties, and prompted a previous paper [17]. This paper will attempt to move beyond the use of strip yield modeling [18] to allow the CTOA fracture criterion greater applicability and provide a theoretical basis for the constant CTOA during fracture process.

A stable crack growth model is proposed in this paper, and this model considers growing crack opening profile as corresponding stationary crack opening profile minus plastic wake profile. Based on this model, the transferability of CTOA was discussed. In Section 2, this model is provided and some further results, such as relations between CTOA and K-R curves// integral, are also presented. In Section 3, the transferability of CTOA was discussed by dimensional analysis and the conclusion that critical CTOA is independent of initial crack length, structural size, and structural configuration for a given material

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