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A simple model to explain transferability of crack tip opening angle

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ABSTRACT

A stable crack growth model based on critical crack tip opening angle (CTOA) is proposed. This model considers the growing crack opening profile as corresponding stationary crack opening profile minus plastic wake profile. A relationship between CTOA and stationary crack tip opening displacement is established. Relations between CTOA and *K*-R curves, *J* integral are then obtained from this relationship and the crack growth model is thus verified. Based on this model, the transferability of CTOA is discussed by dimensional analysis. The critical CTOA is independent of initial crack size, structural size, and structural configuration for a given material and stress state if the plastic zone stress distribution is invariable. A method to predict residual strength based on a critical CTOA is proposed and rudimentary verification is performed using previously published fracture test results.

1. Introduction

As cracks propagate stably in elastic plastic materials, the yielding region is not small and plastic unloading will occur, and then the linear fracture mechanics and elastic plastic fracture parameters based on deformation theory of plasticity will cease to be valid [1]. Crack tip opening angle (CTOA) or displacement (CTOD) at a specified distance from crack tip seems to be most suitable for modeling the stable crack growth case [2].

CTOA reflects the local slope of crack faces near the crack tip and has been found to be nearly constant during stable crack growth [2–4]. In addition, CTOA was found to be independent of in-plane geometry if the crack length and un-cracked ligament size are larger than 4 times thickness [5]. Thus, CTOA was widely used to predict stable crack growth of thinned wall structures, such as stiffened and stiffened with or without multiple sites damage [6–9] during the past three decades. ASTM has also established a standard procedure to measure critical CTOA from C (T) or M (T) specimens [10]. However, a theoretical analysis is needed to explain the constant nature of CTOA when crack length and un-cracked ligament size are large enough. Existing literature explanations [5] are based on incremental finite element analysis and only applied to elastic perfectly plastic materials. Thus, a theoretical analysis to discuss transferability of CTOA is required. Meanwhile, the critical CTOA concept has been used to predict stable crack growth computationally by intensive finite element techniques [2–9]. A simple but reliable engineering method is required to allow the CTOA based fracture criterion applicable to common engineering fracture control issues.

A crack tip opening displacement V_R was defined by Newman [11,12] as the corresponding stationary crack tip opening displacement of a growing crack, and the $V_R - \Delta a$ curve was found to be independent of crack length, specimen width and

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Nomenclature

a	length of crack, mm
a_0	initial length of crack, mm
a_e	effective crack length, mm
B	specimen thickness, mm
C	a parameter depends on specimen types and materials
C_n	a constant depends on strain hardening exponent
d	distance behind the crack tip, mm
d_n	a material constant dependent on materials and stress state
E	Young's modulus, MPa
f, g, G, h, H	function symbols
F	applied load, KN
K	stress intensity factor, MPa$\sqrt{\text{mm}}$
n	strain hardening exponent
(r, θ)	polar coordinates centered at the crack tip
r_p	plastic zone size by Dugdale model, mm
r_y	Irwin plastic zone size, mm
R	scale factor
S	span of SENB specimens, mm
u	crack opening profile, mm
V_R	resistance to crack extension in terms of crack tip opening displacement, mm
W	specimen width, mm
x, x_t	a coordinate centered at the crack tip and with the opposite direction of x_1
(x_1, x_2)	cartesian coordinates
α	material constant
β	constraint factor
Δa	crack extension, mm
δ_0	critical initiation crack opening displacement, mm
δ_a	the advancing crack opening profile, mm
δ_A	crack drive force in terms of crack tip opening profile, mm
δ_c	critical crack opening displacement, mm
δ_H	the plastic wake profile, mm
δ_R	resistance to crack extension in terms of crack tip opening profile, mm
δ_R^M, δ_R^C	$\delta_R - \Delta a$ curves of M (T) and C (T) specimens respectively, mm
δ_R^∞	$\delta_R - \Delta a$ curves of infinite panels
δ_t	the stationary crack opening profile, mm
ε^p	plastic strain
ρ	a length similar to the plastic zone size under small scale yielding condition
σ	applied load, MPa
$\sigma(x)$	stress distributions over the plastic wake region
σ_0	flow stress, $(\sigma_{ys} + \sigma_u)/2$, MPa
σ_u	ultimate tensile strength, MPa
σ_{ys}	yield stress (0.2% offset), MPa
Θ	a generalized dimensionless damage integral
Θ_c	critical value of Θ
ψ_c	critical crack tip opening angle, degrees
CTOA	crack tip opening angle
CTOD	crack tip opening displacement

specimen type. Newman's work and the papers of Deng, Hutchinson and Nilsson et al. [13–16] have generated the idea that the growing crack profile is the superposition of the corresponding stationary crack opening profile and plastic wake profile, providing a direction to study CTOA's properties, and prompted a previous paper [17]. This paper will attempt to move beyond the use of strip yield modeling [18] to allow the CTOA fracture criterion greater applicability and provide a theoretical basis for the constant CTOA during fracture process.

A stable crack growth model is proposed in this paper, and this model considers growing crack opening profile as corresponding stationary crack opening profile minus plastic wake profile. Based on this model, the transferability of CTOA was discussed. In Section 2, this model is provided and some further results, such as relations between CTOA and K-R curves/J integral, are also presented. In Section 3, the transferability of CTOA was discussed by dimensional analysis and the conclusion that critical CTOA is independent of initial crack length, structural size, and structural configuration for a given material

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