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Technical Note

Empirical solutions for stress intensity factors of a surface crack in a solid cylinder under pure torsion

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ABSTRACT

No empirical solution for the stress intensity factors (SIFs) of a surface crack in a solid cylinder under torsion was reported in literature. In this paper, empirical solutions for Modes I, II, and III stress intensity factors of a semi-elliptical surface crack in a solid cylinder under pure torsion are proposed. The dual boundary element method (DBEM) based software was used to generate the SIFs for different crack size, crack aspect ratio, and different inclination angles. Next, the solutions for the SIFs were produced through a systematic curve fitting approach.

1. Introduction

The cylindrical bars are often found in machine components and structures for various applications. Due to applied cyclic loads, material defects and improper manufacturing processes, the components may contain flaws such as surface cracks. Under cyclic loadings, a crack or flaw may propagate into a critical stage, leading to an undesirable shattering failure. Scheduled inspections shall be well planned and performed to ensure the safety of the component to be fit in service. In doing so, understanding of the crack behaviors in components is paramount important. In particularly, quantifying the severity of a surface flaw or crack is a part of developments of the fatigue life prediction. Linear elastic fracture mechanics (LEFM) approach has been widely adopted in engineering design process to particularly evaluate the crack behaviors. The stress intensity factors are the quantitative values commonly used to evaluate the elastic stress-strain field in the vicinity of a crack front. Thus, the evaluation of the stress intensity factors has been a major task in LEFM since decades ago and becomes more essential, especially the possibility of the use of the data in a preliminary design stage.

The SIF solutions for a surface crack in a smooth round bar have been reported by many researchers through theoretical studies, experimental investigations and numerical analyses. James and Anderson [1] described a simple procedure of performing an experimentation to determine the stress intensity factor of a geometry for which no analytical solution is available. However, in doing an analysis of 3D crack problem, the complexities of the experimental setup is greatly concerned. Thus, a numerical analysis is always sought to reduce the experimental work. Raju et al. [2] evaluated the SIFs of a semi-elliptical crack using the three-dimensional finite element method. Carpinteri [3] reported the SIFs of a surface crack in a round bar. Fonte et al. [4] studied the behavior of surface flaws in a round bar subjected to torsion loading. Shih and Chen [5] evaluated the SIFs of an embedded elliptical crack in a round bar

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Nomenclature

a	depth of crack
b	half-diameter of cylinder
c	half-length of crack
d	cylinder diameter
i	cracking modes: I, II, III (Modes)
θ	semi-circular parametric angles
α	inclination angle
Q	shape factor for elliptical crack
τ_{max}	the maximum shearing stress (at the cylindrical surface)
F_w	finite width correction factor
K_i	mode i stress intensity factors
$J_{i\theta}$	curve-fitting functions
$J_{ia\theta}$, $J_{ib\theta}$	sub-curve-fitting functions to $J_{i\theta}$
F_{ia}, F_{ib}	sub- functions to $J_{i\theta}$
$H_{i\theta}$, $G_{i\theta}$	sub-curve-fitting functions to $J_{i\theta}$
$H_{ia\theta}$, $H_{ib\theta}$	sub-curve-fitting functions to $H_{i\theta}$
$G_{ia\theta}$, $G_{ib\theta}$, $G_{ic\theta}$, $G_{id\theta}$ $G_{ie\theta}$, $G_{if\theta}$, $G_{ig\theta}$	sub-curve-fitting functions to $G_{i\theta}$
K_0	normalizing factor for the stress intensity factors
K_{ac}	function used in Mode III, $K_{ac} = 1$ if $a = c$, otherwise $K_{ac} = 0$
L_{ac}	function used in Mode III, $L_{ac} = 0$ if $a = c$, otherwise $L_{ac} = 1$

by introducing collapsed singular elements in the ANSYS finite element software. Shin and Cai [6] evaluated the SIFs of a semi-elliptical surface crack in a rod for different crack aspect ratios by the experiments and numerical modeling using the ABAQUS FEA codes. Ismail et al. [7,8] reported various SIF results for a surface crack in a round bar under bending, torsion and mixed-mode loadings. Predan et al. [9] presented the stress-intensity factors of a semi-elliptical surface crack in a hollow cylinder subjected to torsion that were calculated using a finite-element technique. Carpinteri [10] evaluated the stress intensity factors along the crack front of straight-fronted edge cracks in cylindrical bars subjected to tension or bending loading. A number of works on surface cracks by the boundary element method (BEM) have also been reported in literature. For examples, Mi and Aliabadi [11,12] presented the dual boundary element method for evaluating general 3D crack problems; Cisilino and Ortiz [13] and Ortiz et al. [14] extensively reported the computations of SIFs of three-dimensional crack problems using the BEM. As reported by Cisilino and Ortiz [13], the BEM is found to be preferably suited for quantifying the conservation integrals. This is because the high accuracy of the displacements and stresses including their derivatives at the internal points can be obtained in BEM through their boundary integral representations, as opposed to other numerical techniques, like FEM. For three-dimensional analyses, the conservation integrals are usually used in their so-called surface- or domain-independent form. While adopting the surface-independent form of conservation integrals, the integration can be carried out along a contour in a plane perpendicular to the crack front and also over the surface enclosed by the contour.

However, Anderson and Glinka [15] highlighted that the exact or closed-form solutions, for an example by the weight function method, would greatly reduce the need for finite element or boundary element models for the crack problems. A number of analytical and closed-form solutions for the SIFs of a surface crack have been reported in literature. The exact analytical solutions utilizing the weight function method for calculating the SIFs of a surface crack were reported in [15–19]. Malits [20] presented exact solutions for calculating the SIFs of a circumferential edge crack in a solid cylinder using the dual Dini series equations and a Fredholm integral equation. The explicit equations for the stress intensity factors of different part-through cracks obtained by the curve fitting approaches of the FEM results were presented in [2,5,21]. Newman and Raju [22] proposed an empirical solution for a surface crack in a finite plate subjected to tension and bending loading. Various solutions for stress intensity factors obtained from a variety of methods have also been presented in the compendiums or handbooks [23–25].

Any updates on the closed-form solutions for the stress intensity factors of a part-through crack in solid components are actively sought. This paper presents closed-form solutions for Modes I, II, and III SIFs of a surface crack in a solid cylinder under torsion. A systematic curve fitting approach on the SIFs obtained from the dual boundary element method based-software of BEASY [26] is carried out. BEASY software uses the DBEM which was developed by Mi and Aliabadi [11,12], Cisilino and Aliabadi [27] for treating the crack boundaries, and Rigby and Aliabadi [28] for evaluating the J-integrals. A number of works have reported the accuracy of the DBEM for evaluating the SIFs and having been compared with the available solutions. To name a few, the works were reported in [11,28–30].

2. Closed-form solutions

The schematic diagram of a semi-elliptical surface crack with a crack depth of a and a half of a crack length of c in a solid cylinder is presented in Fig. 1. Modes I, II, and III SIFs of a surface crack in a solid cylinder for the a/d values from 0.025 to 0.1 (on the basis of engineering estimates for considering a small surface crack), the a/c values from 0.333 to 2, and the selected inclination angles α (ranging from 0 to 45° from the transversal plane) are produced by using the dual boundary element method based-software of BEASY [26]. In the DBEM, the discontinuous quadrilateral quadratic elements are used to model the crack surfaces. The use of the discontinuous elements would result in more accurate evaluations of finite-part integrals [11].

As recommended by BEASY [26], in order to have fine meshing for convergence, consideration has to be taken on the mesh grading and aspect ratio. The mesh grading consideration shall be taken to ensure that the size of adjacent elements do not differ greatly. As a general guideline in three-dimensional crack analyses [26], the ratio of the neighboring elements shall not be more than five times its size. The element size on the crack front is set to be around one-fifth of the crack length. Inaccurate results can be expected if the aspect ratio of boundary elements is very large. At the remote region from the crack location, the aspect ratio can be as large as 7:1, i.e. the length is 7 times the width of boundary element. However, the area where the condition changes rapidly, the