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# Discussions on weight functions and stress intensity factors for radial crack(s) emanating from a circular hole in an infinite plate

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ABSTRACT

A thorough verification is made for two recently developed weight functions (WFs) by Jin et al. for a single crack and two symmetric radial cracks emanating from a circular hole in an infinite plate. Through comparisons with other WFs (or Green's functions), it is found that the Jin et al.'s WFs are in marked disagreement to those of Shivakumar-Forman, Newman and Wu-Carlsson, especially for the single crack geometry. Significant disagreements in SIFs obtained by using Jin et al.'s WFs are also found for loadings which are much different from the reference load cases used for deriving their WFs. Possible sources for the poor accuracy of Jin et al.'s WFs are discussed and identified, including limitation of the two-reference-load-cases-based WF approach, choice of reference load cases and sensitivity of these WFs to the inaccuracy of the two reference SIF-solutions. By using a the Wu-Carlsson unified WF approach with crack opening displacement as an additional condition, new analytical WFs for the two hole-edge crack geometries are developed and verified, and further used to derive closed form SIF-expressions for several basic crack line stresses. The new analytical WFs will facilitate more accurate fatigue and fracture analyses for arbitrarily loaded hole-edge crack(s) in an infinite plate.

## 1. Introduction

Radial crack(s) emanating from a circular hole in an infinite plate has received much attention in fracture mechanics research over the past decades. Because of the practical importance in many industries, such as aerospace and petroleum engineering etc., stress intensity factors (SIFs) for the hole-edge crack(s) geometry have been determined for a variety of load cases by using various analytical and numerical methods. However, there are many other more complicated load cases for which solutions are unavailable. The weight function method (WFM), first proposed by Bueckner [1] and Rice [2], provides a very powerful tool for the determination of key fracture parameters such as SIFs and crack opening displacements (CODs) for arbitrary load conditions. Recently, Jin et al. published two papers in this journal on the WFs for a single and double radial crack(s) emanating from a circular hole in an infinite plate: EFM 159 (2016) 144-154 [3] and EFM 170 (2017) 77-86 [4]. Their WF parameters were determined by the method of two reference SIFs developed by Shen-Glinka [5], and were verified by comparisons of SIFs for horizontal/inclined crack(s) under constant/non-constant crack face pressure. Based on the good agreement of SIFs for several load cases in the literature, it was concluded that the Jin et al.'s WFs were "accurate" and "good for nonlinear and complex loadings on crack"[3,4].

Because WF is the property of the crack geometry only (including the composition of traction and displacement boundary composition), and are used for fracture mechanics analyses under *arbitrary* crack face loadings, it is an essential prerequisite that the

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**Nomenclature**

| | |
|---|---|
| $a$ | crack length |
| COD | crack opening displacement |
| CMOD | crack mouth opening displacement |
| $R$ | radius of circular hole |
| $f$, $f_r$ | non-dimensional stress intensity factor, reference non-dimensional stress intensity factor |
| $G(a/R, x/a)$, $G(\alpha, \gamma)$ | Green's function (influence function) |
| $K$, $K_r$ | stress intensity factor, reference stress intensity factor |
| $m(a, x)$, $m(\alpha, \xi)$ | weight function, non-dimensional weight function |
| $M_i$ | Shen-Glinka weight function series coefficients |
| $V_r(\alpha)$ | non-dimensional crack mouth opening displacement for reference load case |
| WF (M) | weight function (method) |
| WCTSE | weight function complex Taylor series expansion |
| $x$ | coordinate along crack with origin at crack mouth |
| $\alpha$ | normalized crack length $\alpha = a/R$ |
| $\beta_i(\alpha)$ | Wu-Carlsson weight function series coefficients |
| $\gamma$ | normalized coordinate $\gamma = x/a = \xi/\alpha$ |
| $\lambda$ | remote tension biaxial ratio |
| $\xi$ | normalized coordinate along crack line with origin ($\xi = x/R = 0$) at crack mouth |
| $\sigma_0$ | scaling factor of crack line stress $\sigma(\xi)$ |
| $\sigma(\xi)$ | crack line stress in a crack-free body |

derived WFs are carefully verified with respect to their true accuracy levels. In the present article, a thorough verification is conducted for the two WFs by Jin et al. [3,4] for hole-edge crack(s) in an infinite plate. Through comparisons with other available WFs (or Green's functions, GFs) for the same crack geometries, it is found that there are marked disagreements between Jin et al.'s WFs and those in the literature obtained by using different approaches, i.e. Shivakumar-Forman[6], Newman[7–9], Wu-Carlsson[10] and Zhao et al. [11], especially for the single hole-edge crack geometry. It is found that the possible sources for the poor accuracy of Jin et al.'s WFs include limitations of the two-reference- load-cases-based WF approach used by Jin et al., choice of the two reference load cases and the sensitivity of their WFs to the inaccuracy of the two reference SIF-solutions.

By using the Wu-Carlsson unified WF approach [10], with crack opening displacement as an additional condition, new analytical WFs for the two hole-edge crack geometries are developed and verified. These new analytical WFs have further improved accuracy and enable the determination of closed form SIF-expressions for several basic crack line stresses, thus will significantly facilitate accurate fatigue and fracture analyses for arbitrarily loaded hole-edge crack(s) in an infinite plate.

## 2. Various weight functions (Green's functions) for hole-edge crack(s) in an infinite plate

The hole-edge crack geometries and the relevant parameters are shown in Fig. 1. Both the single crack ($N = 1$) and symmetric double cracks ($N = 2$) are considered. In the following, analysis is carried out in non-dimensional form, and all the length parameters are normalized by the hole radius $R$, i.e. $\alpha = a/R$, $\xi = x/R$, and $\gamma = \xi/\alpha$. Considering the fact that for $\alpha = a/R \geq 2.0$, the effect of hole on WFs becomes negligibly small and the problem can be simply treated as a center crack in an infinite plate, the present article is limited to $\alpha < 2.0$.Before discussing the WFs, a note is made here on the relationship between WF and the Green's function (GF). The GF represents the SIF ($K$) due to a pair of point forces $P$ (Eq. (1a), for unit thickness, acting at an arbitrary location $x$ at the crack surfaces, as shown in Fig. 1. Because the two functions are related to each other by a very simple factor $\sqrt{\pi\alpha}$ , Eq. (1b), and therefore they are regarded to be equivalent.

$$K = \frac{P}{\sqrt{\pi a}} G(\alpha,\gamma) \tag{1a}$$

$$G(\alpha,\gamma) = \sqrt{\pi\alpha} \cdot m(\alpha,\gamma) \tag{1b}$$

Since the early 1980s, the GFs and WFs for radial crack(s) emanating from a circular hole in an infinite plate have been studied by several researchers using a variety of methods.

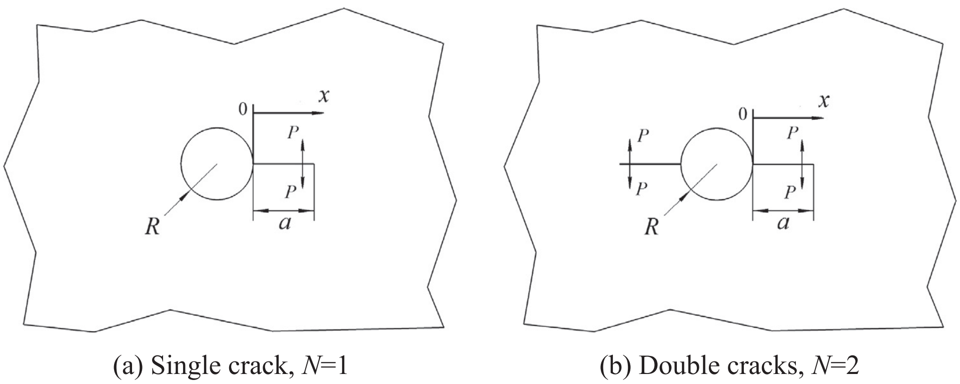


**Fig. 1.** Radial crack(s) emanating from a circular hole in an infinite plate.