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On the distribution and scatter of fatigue lives obtained by integration of crack growth curves: Does initial crack size distribution matter?

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A R T I C L E I N F O

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A B S T R A C T

By integrating the simple deterministic Paris' law from a distribution of initial defects, in the form of a Frechet extreme value distribution, it was known that a distribution of Weibull distribution of fatigue lives follows exactly. However, it had escaped previous researchers that the shape parameter of this distribution tends to very high values (meaning the scatter is extremely reduced) when Paris' exponent m approaches 2, leading to the exponential growth of cracks with number of cycles. In view of the fact that values close to $m = 2$ are of great importance in materials for example used for primary aircraft structures as recognized by some certification requirements (and the so-called "lead crack" methodology), we believe this conclusion may have some immediate relevance for damage tolerance procedures, or certification methods where accurate description of scatter is required. Indeed, we extend the result also to the case when Paris' constant C is distributed, and give also an estimate of the level of scatter expected in propagation life in the most general case when C, m are both random variate alongwith the defect size distribution, based on first transforming them to uncorrelated form C_0, m, and validate this with the famous Virkler set of data. We finally discuss that from known typical values of fatigue life scatter of aeronautical alloys, it is very likely that an important contribution comes from short crack growth.

1. Introduction

The scatter of fatigue properties has been studied for long time but maintains a vivid interest because of the enormous technological implications of maintaining safety in critical applications. Today, design and life management approaches for materials in safety-critical applications typically rely on large testing programs which produce extensive databases of fatigue data: improved understanding of the underlying statistics would help materials engineer to improve materials, testing and certification procedures to become quicker and less expensive, and engineering applications to become more performant, cheaper and lighter because of the extensive use of more advanced probabilistic design. The statistical behavior of fatigue lifetimes has strong similarities with the statistics of brittle materials, and indeed Weibull's distribution was developed by Weibull exactly to describe the scatter of the strength in brittle materials, for which often Weibull shape parameter is considered a "material constant" [1, and references therein]. The classical interpretation of Weibull modulus in terms of a result of possible brittle fracture of the cracks, and the Weakest Link Theory (WLT), connected brittle failure to the theory of extremes values. One example of physically significant distribution functions of extreme values (here, the largest cracks) bears the name of Frechet, having Cumulative Distribution Function (CDF)

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$$P_a(a) = \exp\left[-\left(\frac{a}{\beta_a}\right)^{-\alpha_a}\right] \tag{1}$$

where α_a is a shape parameter, and β_a a scale parameter. By using Griffith equation, such a distribution of crack sizes, under remote uniform tension (assuming no interaction between cracks), can be converted into a distribution of strength which follows the Weibull distribution

$$P_\sigma(a) = 1-\exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{2\alpha_a}\right] = W(2\alpha_a,\sigma_0) \tag{2}$$

where $W(\alpha,\beta)$ stands for the Weibull 2-parameter CDF distribution of shape parameter α and scale parameter β. In other words, the obtained Weibull has shape parameter equal to double of the shape parameter of the original Frechet distribution. We mention these results despite we shall only deal with fatigue, just for introducing the concept of how the "scatter" in defects distribution affects the scatter in strength,[1] although an interesting theory which connects static failure to fatigue, simplifying fatigue as a repeated attempt to cause static failure, has been proposed [2]. Murakami [3] has a very significant amount of work showing mainly the role of size and geometry of small defects on the *fatigue limit*, in particular with simple equations relating the hardness HV and size of defects, or nonmetallic inclusions – whereas our attention will be driven to scatter in fatigue lives far from the fatigue limit. Murakami [3] largely used the Frechet distribution for his rating of (non-metallic) inclusions in steels and in particular in most cases he seems to suggest $\alpha_a = 1$, whereas Wallin [4] collects a few data from the literature in bearing steels or Q&T steels where $\alpha_a = 4$–5. Wang et al. [5] show some support to the fact that the fatigue lives of cast aluminum alloys do follow the "Paris law" prediction for the largest defect size.

Wallin [4] discusses statistical aspects of both the fatigue life distribution, and the endurance limit. However, the emphasis of that paper is on size effects. In particular, he suggests that, using the WLT, high statistical uncertainty or physically non-realistic size effects result from the Gumbel and the Weibull distributions, so he is strongly *in favor of the Frechet distribution*. For the latter, he derives *approximately* that the fatigue life, using a Paris' law for the crack growth, will be described by a Weibull distribution, namely

$$P_{N_f} = W(\alpha_L,\beta_L) \tag{3}$$

where "L" stands for Life, and he seems to suggest two different results for α_L (his eqt. 43 and eqt. 50, neither of which we agree with, and with an important qualitative difference), although we are not able to follow the reason behind this double possibility. Ref. [4] discusses experimental data, but collects data *either* for scatter of fatigue lives, or for inclusion measurements, and not both data from the same case, so we are unable to judge the validity of this result for the scatter of fatigue lives quantitatively, and unfortunately there is no independent numerical validation. The reason why this is not a minor detail but will turn out quite important is that Wallin's result does not give any special significance to the limit case of Paris' constant $m = 2$, or the values close to this, which are extremely important for "damage tolerant" materials in use especially in aerospace applications. We shall derive Wallin's result differently and find a*significant discrepancy*, namely that the coefficient α_L tends to infinity when $m = 2$ and hence leads to extremely small scatter expected in fatigue lives. In other words, the statistics seems to suggest that, no matter how large is the dispersion of the distribution of largest defects in the material, the fatigue life will be almost deterministic. We shall also find the value of β_L, and then we move to generalize the results in the much more general case when Paris' constants are themselves random variate, as it is important to consider especially for short crack growth.

We find in particular that by transforming the distribution of Paris' constants C,m to an uncorrelated form C_0,m a very good and analytical estimate of fatigue life scatter can be done by neglecting the variation in m and validate this result by means of the extensive set of data known in the Literature. We finally discuss some significance of these results, in view of possible deviations from the simple crack growth considerations made here, and we make also general comments about Damage Tolerance and Lead Crack general philosophies.

2. Propagation of cracks with deterministic Paris law

Paris' law [6] relates the crack advancement per cycle to the range of Irwin's stress intensity factors as

$$\frac{da}{dN} = C\Delta K^m \tag{4}$$

where C,m are Paris' "material constants". For many structural materials, m lies in the range from 1.8 to 4, but the value $m = 2$ (which leads to an exponential growth of the crack) which was incidentally suggested in proposals before that of Paris, is attracting more and more interest, especially for lead cracks in aircraft primary structures (Lo et al. [7] and many references therein, [8–10]), so that the United States Air Force (USAF) certification requirement approach to assessing the risk of failure by fracture, makes extensive use of exponential crack growth.[2] We will obtain results both for $m = 2$ or a general $m > 2$, but the significant new results will be

[1] In reality, there are three types of possible distributions of extremes: starting as a parent distribution from the exponential, normal and lognormal distributions, the distribution of largest values will be Gumbel. Starting from power-law tails, we obtain Frechet or Weibull itself.

[2] Certification for damage tolerance requires considering crack growth due to spectrum loading which may involve complex accounts of crack closure which induces generally growth retardation.