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Shear forces, root rotations, phase angles and delamination of layered materials

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ABSTRACT

In this paper, the concepts of root rotations and phase angles in laminated structures are reviewed, with particular reference to how the presence of a shear force affects the fracture mechanics of an interface. It is shown that it is possible to combine all the effects of shear into an effective root rotation, rather than the usual approach of invoking root rotation as a correction to a clamped Timoshenko beam. This simplifies the mechanics of the phenomenon, as root rotation is a general concept for interface cracks, and is not unique to shear loading. The root rotations enter into expressions for the energy-release rate associated with shear, and give rise to the crack-tip phase angles that provide a measure of how other modes of loading interact with shear loading. Additionally, the present analysis identifies the concept of a root displacement as an additional measure of the deformation at a crack tip. This is required to provide consistency with compliance-based approaches to the fracture mechanics of a double-cantilever beam, and allows one to make a direct connection to classical elastic-foundation models of that geometry.

1. Introduction

The complications associated with transverse shear forces applied to layered materials have long been recognized, particularly with regard to the double-cantilever beam (DCB) geometry [1–3]. Conversely, the fracture mechanics for loading by pure moments is very straight-forward: the energy-release rate, $\mathscr{G}$, is dependent only on the applied moments, the elastic properties of the arms, and the thickness of the arms. The energy-release rate, along with the equivalent *J*-integral, and the corresponding work done against the cohesive tractions at the crack-tip, $\mathscr{W}_0$, are not dependent on the nature of the cohesive law for the interface. Experimental approaches have been developed to evaluate interfaces using this concept, with test configurations being designed to ensure loading by pure moments [4,5]. However, more commonly, interfacial properties are measured using a DCB loaded by transverse forces [6,7], so that the crack tip experiences both a moment and a shear force. This introduces the need to consider how shear may affect the interface mechanics.

Historically, shear loading has generally been considered to have two distinct effects in a DCB geometry [1]. First, shear results in an additional contribution to the compliance of the arms themselves. The complications associated with the analysis of this effect have been described by Barber [8] as a problem of satisfying all the conditions at a clamped boundary. Second, the arms of a DCB can rotate at the crack tip, since they are not clamped rigidly. This crack-tip rotation is termed *root rotation*. One contribution to root

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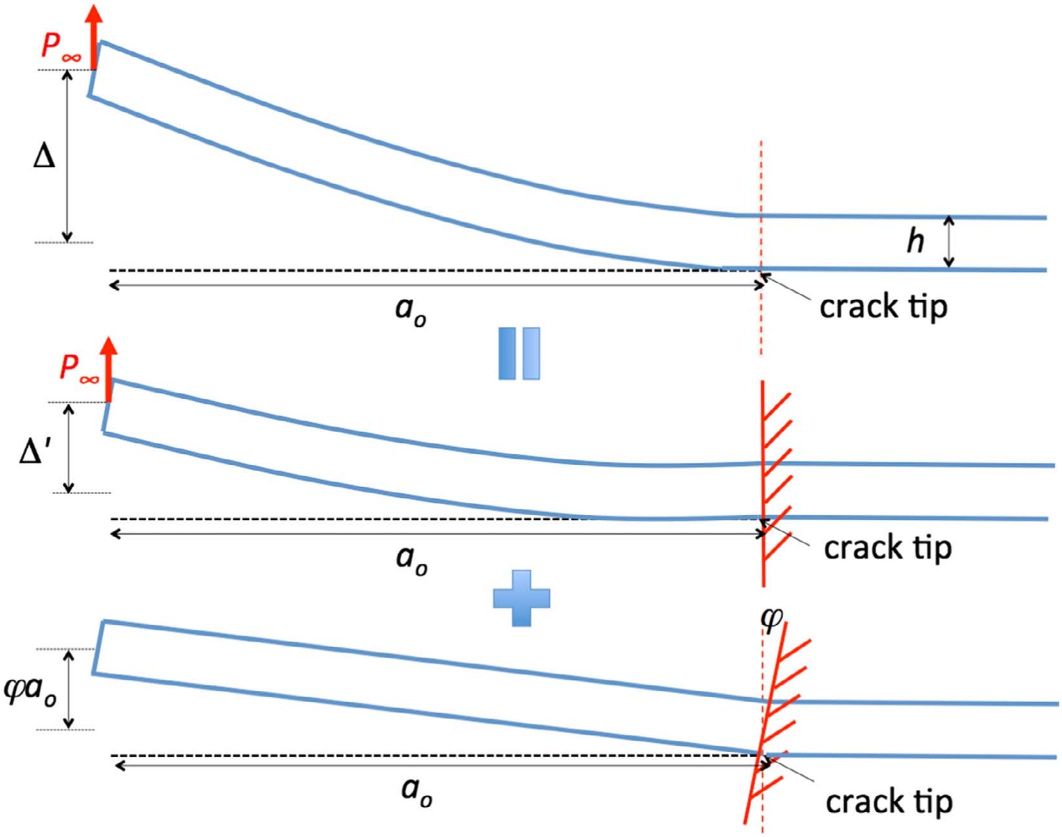

Fig. 1. The crack-mouth opening, Δ of a beam that forms part of a double-cantilever beam with a crack length of a_o (top), can be considered to be made up of two contributions: $\Delta' = 4P_\infty a_o^3/Eh^3 + P_\infty a_o/\kappa_s Gh$, representing the deflection of a clamped Timoshenko beam (middle), and φa_o, representing the rotation of the clamped boundary (bottom).

rotation arises even in perfectly bonded systems,[1] because the elastic arms are compliant beyond the crack tip. A second contribution arises if the bonding along the interface results in additional compliance.

As an illustration of the effects of shear, consider the displacement of a point force (per unit width), P_∞, located at a distance a_o from the crack tip in a symmetrical, perfectly bonded, linear-elastic double-cantilever beam whose arms have a Young's modulus of E, shear modulus of G, and thickness of h (Fig. 1). The displacement of the force on each arm is given by [1]

$$\Delta = \frac{4P_\infty a_o^3}{Eh^3} + \frac{P_\infty a_o}{\kappa_s Gh} + a_o\varphi, \tag{1}$$

where κ_s is the shear coefficient, which is often taken to be 5/6, and φ is the root rotation. In this expression, the first term is the result assuming an Euler-Bernoulli beam, the second term is the shear correction for a clamped Timoshenko beam, and the third term represents the root-rotation correction.

Calculating the displacement from Eq. (1) then becomes a matter of determining φ as a function of geometry and loads. Once the displacement is known, the potential energy of the arms can be determined, from which $\mathscr{G}$ can be calculated. Early work used approximate assumptions to model the deformation at the crack tip. These gave results that, while usefully accurate, were not rigorously correct. For example, Gillis and Gilman [1] assumed a simple power-law (quadratic) dependence between φ and the crack length. Mostovoy et al. [2] assumed that the effect of root rotation could be modelled by changing the effective crack length. The notion of an effective crack length was further refined [10], and forms the basis of current standards for DCB tests [6,11].[2] More formally, magnitudes of the root rotations have been determined numerically [12], and from approximate analytical analyses [13,14]. The corrections associated with root rotations have also been determined empirically for specific test geometries [15,16].

An alternative approach for calculating the effect of shear in laminated geometries has been provided by Li et al. [17] for linear systems. In this approach, any general loading is expressed as the superposition of four fundamental crack-tip geometries shown in

[1] The term "perfect bonding" is used here to describe an interface having a traction-separation law with an infinitely steep loading slope, corresponding to a cohesive length of zero [9]. Any compliance associated with the traction-separation law results in a finite cohesive length [9].

[2] The ASTM standard for adhesive bonding [7] uses the basic results of Mostovoy et al. [2], with their approximation that $\kappa = 2/3$ and $\nu = 1/3$, but with no suggested modification for root-rotation effects.