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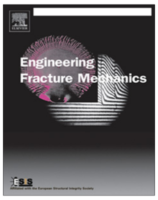

# Estimation of $C^*$ including the effect of threshold stress

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## ABSTRACT

In some alloys such as 9%Cr heat resistant steels and magnesium alloys, the creep constitutive equation of the power-law requires a term of threshold stress due to the presence of second phase particles. It is necessary to establish an estimation method of $C^*$ for such alloys to predict the life of their components. In this paper, the General Electric/Electric Power Research Institute (GE/EPRI) method and the reference stress method were modified to estimate $C^*$ for power-law creep materials with threshold stress. The finite element method was used to verify the accuracy of the modified methods. The accuracy of the calculation equation of $C^*$ in the American Society for Testing Materials (ASTM) E 1457 was also assessed. The results indicated that the modified GE/EPRI method was sufficiently exact as an engineering method. $h_1$ was slightly affected by the applied load and significantly affected by the threshold stress. The accuracy of the modified reference stress method increased with increased applied load and was within ±40%. The accuracy of the calculation equation of $C^*$ in ASTM E 1457 was not affected by the threshold stress and the equation could be directly used for power-law creep materials with threshold stress.

## 1. Introduction

$C^*$ is one of the parameters used for characterizing the creep crack growth and the stress field at the crack tip region. Accurate estimation of $C^*$ plays an important role in the analysis of fracture mechanics and lifetime prediction for structures at elevated temperatures. Many studies have been conducted to research estimation methods of $C^*$. The General Electric/Electric Power Research Institute (GE/EPRI) method is widely used to estimate $C^*$ for homogeneous materials [1,2]. The reference stress method is another widely used method for estimating $C^*$ [3,4]. Based on the reference stress method, Xuan [5] proposed a method to estimate $C^*$ for mismatched weld creep cracks and Kim [6,7] proposed an enhanced reference stress method to estimate $C^*$. Other methods have also been proposed to estimate $C^*$ for mismatched weld creep cracks [8,9] in addition to cracks in thin T-sections [10] and annular discs [11]. Further, the calculation equation of $C^*$ in the American Society for Testing Materials (ASTM) E 1457 was modified for mismatched weld creep cracks by Xuan [12]. However, there has been no research on the estimation of $C^*$ for materials with power-law creep constitutive equations that include a term of threshold stress. Since threshold stress exists in some alloys due to the presence of second-phase or nanometer-sized particles, such as dispersion hardened alloys [13–16] and nanocomposites [17,18], it is necessary to establish estimation methods of $C^*$ with the effect of threshold stress included.

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**Nomenclature**

| | |
|---|---|
| $A$ | power-law creep coefficient |
| $a$ | crack length |
| $B$ | specimen thickness |
| $C^*$ | a path-independent $C$-integral defined under the extensive steady state creep stage |
| $C^*_{\mathrm{EPRI}}$ | $C^*$ estimated by the GE/EPRI method |
| $C^*_{\mathrm{FEM}}$ | $C^*$ calculated by finite element method |
| $C^*_{\mathrm{REF}}$ | $C^*$ estimated by reference stress method |
| $C^*_{\mathrm{Vc}}$ | $C^*$ calculated by the equation in ASTM E 1457 |
| $E$ | Young's modulus |
| $e_{\mathrm{EPRI}}$ | $e_{\mathrm{EPRI}} = 100\% \times (C^*_{\mathrm{EPRI}} - C^*_{\mathrm{FEM}})/C^*_{\mathrm{FEM}}$ |
| $e_{\mathrm{REF}}$ | $e_{\mathrm{REF}} = 100\% \times (C^*_{\mathrm{REF}} - C^*_{\mathrm{FEM}})/C^*_{\mathrm{FEM}}$ |
| $e_{\mathrm{Vc}}$ | $e_{\mathrm{Vc}} = 100\% \times (C^*_{\mathrm{Vc}} - C^*_{\mathrm{FEM}})/C^*_{\mathrm{FEM}}$ |
| $\dot{\varepsilon}$ | creep strain rate |
| $\dot{\varepsilon}_{\mathrm{ref}}$ | creep strain rate at $\sigma_{\mathrm{ref}}$ |
| $h_1(a/W, n)$ | dimensionless function of $a/W$ and $n$ |
| $h_1(a/W, n, \sigma_0)$ | dimensionless function of $a/W$, $n$, and $\sigma_0$ |
| $n$ | power-law creep exponent |
| $\eta_1$ | a dimensionless function dependent on $a$ and $W$ |
| $P$ | applied load |
| $P_{\mathrm{L}}$ | plastic limit load |
| $(\dot{V}_{\mathrm{c}})_{\mathrm{SS}}$ | load line deflection rate under the extensive steady state creep stage |
| $W$ | specimen width |
| $v$ | Poisson's ratio |
| $\sigma, \dot{\sigma}, \sigma_0$ | stress, stress rate, and threshold stress, respectively |
| $\sigma_{0.2}, \sigma_{\mathrm{ref}}$ | 0.2% proof stress or the stress at 0.2% inelastic strain, and reference stress, respectively |

In this paper, the GE/EPRI method and the reference stress method were implemented first, then modified to estimate $C^*$ for power-law creep materials with threshold stress. The finite element method was then used to verify and assess the accuracy of the modified methods. The accuracy of the calculation equation of $C^*$ in ASTM E 1457 was also investigated.

## 2. Estimation method of $C^*$

When the primary creep and tertiary creep stages are ignored, the steady state creep stage with elastic properties is usually described by a constitutive equation of elastic plus power-law creep [9]:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + A\sigma^n \tag{1}$$

where $A$ and $n$ are the creep coefficient and exponent, respectively; $E$ is the elastic modulus; $\dot{\varepsilon}$ is the creep strain rate; $\sigma$ and $\dot{\sigma}$ are the stress and stress rate, respectively. Under the extensive steady state creep stage, $C^*$ for compact tension (CT) specimens can be estimated using the GE/EPRI method (denoted as $C^*_{\mathrm{EPRI}}$):

$$C^*_{\mathrm{EPRI}} = A(W - a)h_1(a/W, n)\left(\frac{P}{1.455\eta_1 B(W - a)}\right)^{n+1} \tag{2}$$

where $W$ is the specimen width, $B$ is the specimen thickness, $a$ is the crack length, $P$ is the applied load, $\eta_1$ is the dimensionless function of $a$ and $W$, and $h_1$ is the dimensionless function of $a/W$ and $n$ [19]. $C^*$ can also be estimated using the reference stress method (denoted as $C^*_{\mathrm{REF}}$) [20]:

$$C^*_{\mathrm{REF}} = \left(\frac{K^2}{E'}\right)\frac{E\dot{\varepsilon}_{\mathrm{ref}}}{\sigma_{\mathrm{ref}}} \tag{3}$$

where $K$ is the stress intensity factor, $\sigma_{\mathrm{ref}}$ is the reference stress, $\dot{\varepsilon}_{\mathrm{ref}}$ is the creep strain rate at $\sigma_{\mathrm{ref}}$ ($\dot{\varepsilon}_{\mathrm{ref}} = A\sigma^n_{\mathrm{ref}}$), $E' = E$ for plane stress, and $E' = E/(1 - v^2)$ for plane strain. In addition, based on the ASTM E 1457 standard [21], $C^*$ for CT specimens can be calculated with the following equation using the load line displacement rate (denoted as $C^*_{\mathrm{Vc}}$):

$$C^*_{\mathrm{Vc}} = \frac{P(\dot{V}_{\mathrm{c}})_{\mathrm{SS}}}{BW}\left(\frac{2}{(1 - a/W)} + 0.522\right)\frac{n}{n+1} \tag{4}$$

where $(\dot{V}_{\mathrm{c}})_{\mathrm{SS}}$ is the load line deflection rate under the extensive steady state creep stage.

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