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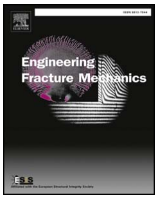

A wide range stress intensity factor solution for an eccentrically cracked middle tension specimen with clamped ends

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ABSTRACT

A wide range K_I solution has been developed for an eccentrically cracked middle tension specimen with clamped ends. In the derivation of the K_I solution, an equivalent model was developed; weight functions for eccentric cracks in a finite-width strip were utilized, with approximate closed form K_I and plate-end displacement solutions obtained. Good agreements between the approximate solutions and corresponding FEM results were observed over a wide range of eccentricities and crack lengths. The accuracy of the K_I solution was analyzed under different aspect ratio conditions, with the applicability of the approximate solutions given. The equivalent model was also discussed.

1. Introduction

Of the standard specimen types provided in Ref. [1], the middle tension (M(T)) specimen, one of the most widely used specimen for fatigue-crack-growth-rate testing in laboratory, is the only one applicable to both tension-tension and tension-compression loading conditions. To ensure the accuracy of the stress intensity factor solution in Ref. [1], a crack symmetry requirement has been specified for the M(T) specimen, stating that measurements referenced from the specimen centerline to the two crack tips will differ by up to 0.025 W (W is the specimen width). However, asymmetric crack growth may occur due to the following reasons:

- Notch quality: When using milling, electrical-discharge machining or saw cutting in notch preparations, notches at both sides may not share the same quality, e.g. notch root radius. Consequently, the crack initiation time will be influenced, resulting in different crack lengths at both sides after fatigue precracking;
- Material-related problems: Material inhomogeneity, such as inclusions in metallic materials, may lead to sudden change in crack growth rate, and thus asymmetric crack growth.

Currently, new materials and processing technics have constantly emerged, such as diffusion-bonded metallic laminate and metal additive manufacturing. An investigation on the crack growth behavior of diffusion-bonded laminates of titanium alloy [2] showed that asymmetric crack growth inevitably occurred when using M(T) specimens fabricated from the titanium alloy laminates. According to Ref. [1], a significant portion of fatigue crack growth data proves invalid, thus resulting in time and resources waste. As a result, there is an urgent need for data processing techniques for fatigue crack growth data that fails to meet the crack symmetry requirement. The core of the data processing techniques is the calculation of stress intensity factors for the eccentrically cracked M(T) specimen under laboratory conditions. In laboratory, the M(T) specimen is usually gripped by the friction grips of the testing

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Nomenclature

a	half-crack length
a_A	coordinate of crack tip A
a_B	coordinate of crack tip B
C	coefficient in Paris-Erdogan law
e	coordinate of the crack center
E	elasticity modulus
$F^{A,B}$	correction factors for crack tip A and B
H	half-gage length
K_I	mode I stress intensity factor
$K^{A,B}$	K_I for crack tip A and B
$K_M^{A,B}$	K_I for crack tip A and B under pure bending
$K_{\sigma_0}^{A,B}$	K_I for crack tip A and B under uniform tension
M	bending moment
n	exponent in Paris-Erdogan law
N	number of load cycles
P	remote tensile force
u	plate-end displacement along x axis
U	energy added to produce crack extension from zero to $2a$
v	plate-end displacement along y axis
v_g	normalized plate-end displacement
V	energy of the cracked plate
V_0	strain energy of the uncracked plate
W	half-plate width
ε	normalized eccentricity
θ	plate-end rotation
λ	normalized crack length
σ_0	uniform tensile stress
ΔK	stress intensity factor range
FEM	finite element method
M(T)	middle tension
$m^{A,B}(x,a_A,a_B)$	weight functions for crack tip A and B
$\sigma(x)$	stress distribution along $y = \pm$H
$\sigma_1(x)$	stress function supplying additional bending moment
$\sigma_2(x)$	stress function supplying tensile force

machine, and axial load is applied to the specimen. Rotations and lateral displacements of the specimen end are constrained, while axial displacements are allowed. Such a condition will be referred to as the clamped end condition.

Under the clamped end condition, the stress intensity factors are almost the same as those under uniform tension for symmetric cracks in the M(T) specimen as long as the plate aspect ratio (i.e. the ratio of plate length to width) is larger than 1.5 [3]. However, when eccentric cracks emerge, the bending moment associated with the rotational constraint will influence the stress/strain fields near the crack tips. Thus, the stress intensity factors will differ from those under uniform tension. Therefore, stress intensity factor (hereafter called K_I) solutions need to be developed for the eccentrically cracked M(T) specimen under the clamped end condition.

The M(T) specimen can be simplified as a plate. Thus far, fewer investigations have been conducted on the K_I for an eccentrically cracked plate. Only Terada and Isida [4] conducted pertinent studies. Terada and Isida [4] obtained the K_I solution for an eccentrically cracked plate under uniform tension through stress analysis. However, due to the difficulty in the implementation of analytical methods for the determination of the unknown coefficients in the stress function, closed form K_I solutions were not provided. Therefore, the K_I solution provided in Ref. [4] has been rarely employed in engineering applications. With respect to the K_I for the tension of an eccentrically cracked finite-width strip, pertinent investigations are relatively abundant. Of all those investigations, Isida [5,6] obtained approximate, albeit highly accurate, K_I solutions by means of Laurent series expansion of the complex stress potentials. Cartwright and Rooke [7], Gray [8] and Wang [9,10] obtained approximate K_I solutions with other methods including compound method [7] and crack-line stress field method [9,10]. The error of the K_I solutions provided in Refs. [7–10] is larger than that of Isida's solution [5,6]. Taking uniform tension as the reference load case and using Isida's solution [5,6], Chen and Albrecht [11] assumed a crack-opening displacement function and derived weight functions for eccentric cracks in a finite-width strip. By performing power series expansion of the crack opening displacement field, Fett [12,13] and Ng and Lau [14] gave three other sets of weight function expressions.

It should be noted that the K_I solutions in Refs. [5–10] were developed for eccentric cracks in a strip under uniform tension without displacement constraints, distinguishing it from the clamped end condition. Moreover, for eccentric cracks in a plate with clamped ends, K_I depends on the plate aspect ratio, which is not the case for a plate with unconstrained ends. Though weight functions were provided for eccentric cracks in a strip [11–14], they were obtained based on the K_I solution for an eccentrically cracked strip under uniform tension involving no displacement boundary conditions. Wu and Carlsson [15] indicated that weight functions depend not only on the geometry of the cracked body, but also on the nature of boundary conditions, i.e. the boundary distribution of traction and displacement loading. The clamped end condition belongs to the mixed boundary condition due to the combined action of remote tensile force and constraints on plate end rotation and displacement. Besides, the plate-aspect-ratio effect is not reflected in the weight functions for a strip. Therefore, the weight functions provided in Refs. [11–14] are not applicable to an eccentrically cracked plate with clamped ends.

For asymmetric crack growth associated with the M(T) specimen, Ref. [16] indicated that Isida's solution [5,6] can be used to calculate K_I for the M(T) specimen with clamped ends when the crack center and crack length fall into a specific range. When out of that range, Isida's solution [5,6] could produce large errors. Finite element method (FEM) is powerful and can be used to calculate K_I for an eccentrically cracked plate with clamped ends. However, the non-parameter nature and time-consuming characteristic of FEM impose restrictions on its frequency use in stress intensity factor calculations, especially data processing of fatigue crack growth data. Therefore, an efficient solution applicable to a wide range of eccentricities and crack lengths needs to be developed.

When the specimen is gripped by the friction grips, the specimen deformation inside the grips will be fully restricted. The Poisson effect is consequently restrained. In contrast, the specimen between the grips, i.e. the gauge section, is allowed to deform. As a result, the stress/strain varies drastically near the gauge-section boundary, adding difficulty to find an appropriate stress/strain function