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A phenomenological approach to fatigue with a variational phase-field model: The one-dimensional case

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ABSTRACT

We propose a new variational fatigue phase-field model. The basic idea of the model is to let the fracture energy decrease as a suitably defined accumulated strain measure increases, which is obtained by introducing a dissipation potential which explicitly depends on the strain history. This amounts to a phenomenological description of a multitude of microscopic material degradation mechanisms, that are responsible for the macroscopic evidence of fatigue effects.

In this first step of a longer term project, the analysis is limited to the simplest possible setting, namely: linear elasticity, brittle material behavior, symmetric response in tension and compression. However, in this variational framework, the extension to include additional material phenomena, such as plasticity, is straightforward. We show the results of numerical simulations based on a solution strategy devised from the variational approach. Already with the choice of simple constitutive functions, based on few key constitutive parameters, the present model is capable to describe typical $\sigma - N$ (Wöhler) curves, to recover the known trends in the description of the mean-stress effects and to account for generic loads in a straightforward manner.

1. Introduction

The formation and propagation of cracks due to fatigue is one of the critical causes of failure in many engineering structures, and its prediction still represents a challenge for modeling and simulation. Fatigue of materials, caused by cyclic loadings, can be described as having several stages. In the initial stage, irreversible microscopic degradation phenomena take place in the material (*e.g.* micro-cracks, plastic slip systems), [44, Section 4, 5 and 6]. Damage is cumulated as the application of loads continues; hence a macroscopic crack appears which eventually starts to propagate. This fatigue degradation process gradually reduces not only the material stiffness but also the amount of energy per unit volume that a material can absorb before further rupturing.

The details of the phenomenology are very complex and obviously influenced by the specific properties of the material at hand, including its microstructural features [42].

The resistance against fatigue is generally expressed in terms of number of cycles or time prior to failure (fatigue life) and mostly quantified by statistical methods, approaches based on fracture mechanics or constitutive material models including

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Nomenclature

D	half damage localization support width
$\mathcal{D}$	total dissipated work
d	fatigue degradation function
$\mathcal{E}$	total internal potential energy
E	damage dependent elastic modulus
f	fatigue function
f_{d}	damage/fatigue threshold function
f_r	reaction force
L	bar length
$\mathcal{L}$	total external work
ℓ	internal material length
L^*	critical bar length leading to a snap-back response
N	number of load cycles
N_f	number of load cycles at which fracture occurs
S	damage dependent compliance function
t	time-evolution parameter
Δt	half-period of a load cycle
$\bar{u}$	prescribed displacement
u	displacement
w	damage dissipation function
$\bar{x}$	damage localization center
α	damage
ε_{a}	strain amplitude
ε_{d}	damage strain
ε_{e}	strain endurance limit
ε_{m}	mean strain
ε_{f}	fatigue strain
ε	infinitesimal strain
$\bar{\varepsilon}$	accumulated strain
ε_{y}	threshold strain
φ	dissipation potential
ψ	internal potential energy density
σ_{d}	damage stress
σ_{e}	stress endurance limit
σ	stress
σ_{y}	threshold stress
σ_{f}	fatigue stress

fatigue effects. Statistical methods are based on the analysis of a large number of experimental tests reproducing the specific situation at hand and hardly allow for generalizations [27]. The most popular correlation relationship is based on the so-called Wöhler curve that relates the stress (strain) amplitude of a cyclic load to the corresponding fatigue life. Fracture mechanics based approaches typically adopt the Paris theory, relating the variation during the loading cycles of the energy release rate with the growing rate of the fatigue crack by means of experimentally calibrated parameters [40,35]. This approach is valid only in the so-called Paris regime, i.e. away from the transient phases of fatigue crack propagation (crack nucleation and right before failure). Material models based on the definition of suitable constitutive laws including fatigue effects are the most versatile approach and are typically also rooted in fracture mechanics.

Several authors have tied the fatigue phenomenon to the progressive growth of cohesive fractures, see e.g. [46,35,3,30,17] for numerical approaches and [25,26,2,1,19] for some closed form analytical results. These approaches are very elegant from the mathematical viewpoint as they prove that the fatigue phenomenon is already included in sufficiently “rich” constitutive models based on fracture mechanics. They require, at each load cycle, the explicit resolution of the near-tip plastic and strain fields to compute the actual accumulation of the crack opening.

We adopt here a more pragmatic approach and prefer not to rely on the solution of the micro-scale problem but rather to focus on the macroscopic effects of the fatigue phenomenon, in the spirit of phase-field approaches. We propose a variational phase-field model of fracture including a phenomenological description of fatigue effects, focusing for the moment on a one-dimensional formulation and on brittle fracture with symmetric behavior in tension and compression.

The phase-field approach to fracture [15,12] is recently gaining a lot of attention, see [10] for a review. Its main advantage is the possibility of modeling arbitrarily complicated crack patterns through the solution of partial differential equations. Phase-field models have a variational structure and do not need ad hoc criteria for topological changes in the crack pattern (branching or merging, [29,14]). The state of the material is characterized by the crack phase-field (or simply phase-field)

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