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The moving least squares based numerical manifold method for vibration and impact analysis of cracked bodies

Wei Li [a,c], Hong Zheng [b,*], Guanhua Sun [a,c]

[a] State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan 430071, China
[b] Key Laboratory of Urban Security and Disaster Engineering (Beijing University of Technology), Ministry of Education, Beijing 100124, China
[c] University of Chinese Academy of Sciences, Beijing 100049, China

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ABSTRACT

In the numerical manifold method (NMM), the mathematical patches composing the mathematical cover can take on any shape in addition to finite elements. In this study, the influence domains of those scattered nodes in the moving least squares (MLS) approximation serve as the mathematical patches. And accordingly, the MLS based NMM, designated by MLS-NMM, is derived, which owns the advantages of both NMM and MLS. Then, MLS-NMM is applied to analyze cracked bodies under dynamic loading, and a mass lumping scheme fit for MLS-NMM is proposed. The dynamic analyses of some typical cracked bodies are carried out, indicating MLS-NMM equipped with the proposed mass lumping scheme has more excellent numerical properties than that with the consistent mass formulation in the analysis of the time domain as well as the frequency domain.

1. Introduction

Over the past few decades, the numerical simulation methods for dynamic crack problems have had many significant developments, among which are the boundary element method [1,2], the finite element method (FEM) [3,4], the meshfree method [5–7], the extended finite element method [8–10], the generalized finite element method (GFEM) [11], the cracking-particle method [12,13], the peridynamics [14,15], to name a just few. Compared with all these methods, the numerical manifold method (NMM) [16] has its own advantages, especially for the analysis of cracked bodies.

On account of the separation of the mathematical cover and the physical cover, NMM is capable of simulating continuous and discontinuous problems in a unified way. Meanwhile, the best approximation of the primal variables can be sought, because high quality meshes that are nearly uniform can always be used to construct the mathematical cover [17]. NMM is also regarded as a substantial improvement of FEM and generalization of the discontinuous deformation analysis (DDA) [18,19]. NMM does not need to remesh in the simulation of crack propagation, has the same ability to treat contact [20] as DDA, the distinct element method [21] and the distinct lattice spring model [22].

The implementation of the p-adaptive analysis is easy in NMM by using higher degree polynomials or special function series as local approximations. In the early days, Chen et al. developed the high-order NMM [23]. Similar to GFEM, the issue of linear dependency might appear. An et al. proposed an algorithm for predicting the rank deficiency of the stiffness matrix [24]. Ghasemzadeh et al. proposed a scheme with high order polynomials as the local approximations that have no linear

* Corresponding author.
E-mail address: hzheng@whrsm.ac.cn (H. Zheng).

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Nomenclature

$\boldsymbol{a}_i$	8-dimensional time dependent vector associated with a singular patch
A_I	small area around a crack tip in calculation of stress intensity factor
$\boldsymbol{b}$	body force per unit volume
$\boldsymbol{B}_k$	10×10 orthonormal matrix
$\boldsymbol{d}$	degrees of freedom of all the physical patches
$\boldsymbol{D}$	elasticity matrix
c_k, s_k	first two items of Williams' displacement series
e	relative energy error
E	Young's modulus
$\boldsymbol{E}_i$	2×8 matrix with four enrichment functions
$\boldsymbol{f}$	global nodal force vector
h	space between two mathematical nodes
$\boldsymbol{I}_2$	2×2 identity matrix
$I^{(1,2)}$	dynamic interaction integral
$\boldsymbol{K}$	global stiffness matrix
K_I^{dyn}	dynamic stress intensity factor corresponding to mode I
K_{II}^{dyn}	dynamic stress intensity factor corresponding to mode II
m_i	the number of physical patches generated from mathematical patch M_i
$\boldsymbol{M}$	global mass matrix
$\boldsymbol{M}_L$	lumped global mass matrix
M_i	the ith mathematical patch
$\boldsymbol{n}$	unit outward normal
n^M	the number of the mathematical patches
n^P	the number of the physical patches
$\boldsymbol{N}$	shape function matrix
$q(x, y)$	bounded weighting function in calculating stress intensity factor
P_{i-j}	the jth physical patch generated from M_i
P_i	the ith physical patch
(r, θ)	polar coordinate
R	radius of the influence domains of nodes
R_d	scaling factor determining the length of area A_I
t	time
$\bar{\boldsymbol{t}}$	prescribed traction vector
$u_i^{(1)}$	actual displacements
$u_i^{(2)}$	auxiliary displacements
$\boldsymbol{u}_i$	2-dimensional time dependent vector of physical patch i
$\boldsymbol{u}^h$	global displacement approximation
$\boldsymbol{u}_i^h$	local displacement approximation of physical patch i
$\ddot{\boldsymbol{u}}$	acceleration vector
$\bar{\boldsymbol{u}}$	prescribed displacement vector
$\dot{\boldsymbol{u}}$	initial velocity field
$W^{(1,2)}$	interaction strain energy
(x, y)	rectangular coordinates
$\boldsymbol{z}$	$= (x, y)$
α, β, ξ	parameters for time integration scheme
Γ_u	displacement boundary
Γ_t	traction boundary
Γ_c	crack surface
δW_I	virtual work of inertia force
Δt	time increment
Δt_c	critical time step
Δt_c^{NL}	critical time step using lumped mass matrix with no crack
Δt_c^{NC}	critical time step using consistent mass matrix with no crack
Δt_c^L	critical time step using lumped mass matrix with crack
Δt_c^C	critical time step using consistent mass matrix with crack
$\Delta \bar{t}_c^L$	normalized critical time step corresponding to lumped mass matrix
$\Delta \bar{t}_c^C$	normalized critical time step corresponding to consistent mass matrix
$\boldsymbol{\Lambda}_k$	10×10 diagonal matrix
$\boldsymbol{\varepsilon}$	strain tensor

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