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Two collinear mode-III cracks in one-dimensional hexagonal piezoelectric quasicrystal strip

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This paper studies two collinear permeable anti-plane shear or mode-III cracks of equal length lying at the mid-plane of a one-dimensional hexagonal piezoelectric quasicrystal strip. By using the integral transform technique, the associated mixed boundary value problem is reduced to triple integral equations, and then to a singular integral equation. By making transformation of variables, a standard singular integral equation with Cauchy kernel of the first kind is derived and exact closed-form solution is obtained. Explicit expressions for the electroelastic field including phonon and phason stresses, electric field and crack tearing displacements at crack faces are determined. The formulae for calculating field intensity factors and energy release rate at the inner and outer crack tips are given, respectively. Some special cases of the present results for the field intensity factors and the energy release rate are discussed. Effects of material properties and geometric size on fracture parameters are examined.

1. Introduction

As a kind of new solids, quasicrystal (QC) discovered by Shechtman et al. [1] has aroused much interest in their physical, chemical and structural properties. QC has long-range orientational order and long-range quasiperiodic translation order, which are different from conventional crystalline materials and non-crystalline materials [2]. Due to quasi-periodic lattice structure, QC has many special properties, such as low friction coefficients, low adhesion, high wear resistance and low porosity [3]. On this account, QC can be used in coating surface of engines, solar cells, thermoelectric converters and containers of nuclear fuel. Since the discovery of QC, QC has become an important branch of physics of condensed material.

Since a quasi-periodic structure may be taken as a projection of higher-dimensional periodic lattice to the physical space, in addition to the usual physical or parallel space described by phonon field, its description needs to introduce a complementary or perpendicular space described by phason field. Therefore, for the phonon field, the usual displacement vector u_j and stress tensor σ_{ij} in the parallel space are used, while for the phason field, a new displacement vector w_j and stress tensor H_{ij} in the perpendicular space are similarly introduced. Thus the total displacement $\mathbf{u} \oplus \mathbf{w}$ lies in a higher-dimensional space [4]. In such manner, the generalized theory of elasticity of QC was developed by Ding et al. [5,6].

Defects in QC such as dislocations and cracks have been observed in experiment. In theory, a crack embedded in QC has been analyzed within the framework of the continuum mechanics approach by Li et al. [7], who investigated a Griffith crack

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Nomenclature

a	the half spacing between the inner tips of two cracks
c	the half spacing between the outer tips of two cracks
C_{44}	phonon elastic modulus
d_{15}	phason piezoelectric constant
D_i	electric displacement components
e_{15}	phonon piezoelectric constant
E_0	stands for $-\Phi_0/h$
E_i	electric field components
$F(\lambda, k)$	the incomplete elliptical integral of the first kind
G_a, G_c	energy release rate at inner and outer crack tips
H_{ij}	phason stresses
K_a, K_c	intensity factor at the inner and outer crack tips
K_2	phason elastic modulus
R_3	phonon-phason coupling modulus
u_j	phonon displacements
w_j	phason displacements
β	stands for $\pi/2h$
ε_0	stands for u_0/h
ε_{ij}	phonon strains
ϕ	electric potential
Φ_0	constant electric potential
η_0	stands for w_0/h
λ_{11}	permittivity
$\Pi(\lambda, r, k)$	the incomplete elliptical integral of the third kind
ω_{ij}	phason strains
σ_{ij}	phonon stresses

problem in a pentagonal QC and obtained analytical expressions for the stress field and stress intensity factors near the crack tips. Later, further studies have been conducted in this field [8,9]. Complex potential method was also extended to treat static crack problems in QC [6]. Gao et al. [10] coped with plane problems of cubic QC with an elliptic hole or a crack. Recently, crack path prediction and crack deflection in one-dimensional (1D) QC were investigated theoretically within a quasi-static framework [11]. The generalized Stroh formalism of 1D piezoelectric QC was applied to develop Green's function of two-dimensional decagonal QC for forces, dislocations and charges [12].

Piezoelectricity is an important physical property of QC, and it has been studied from a theoretical viewpoint [13,14]. Taking advantage of group theory, Hu et al. [4] and Rao et al. [14] investigated piezoelectric effects in both phonon and phason fields. Grimmer [15] derived the piezoelectric constants of second order for QC with any symmetry. For 1D QC, Li and Liu [16] deduced all material properties including thermal expansion coefficient tensors, piezoelectric coefficient tensors and elastic constant tensors under 31 point-groups for the 1D quasicrystals. Altay and Dökmeci [17] used variational principle to derive 3D fundamental equations of elasticity of QC with piezoelectric effect. Due to the piezoelectric effect, QC is expected to be exploited as sensor and actuator in intelligent structures and systems like conventional piezoelectric crystals.For 1D hexagonal QC with piezoelectric effect, Guo and co-workers dealt with a three-phase cylinder model [18] and an elliptical inclusion problem [19]. Li et al. derived a fundamental solution for 1D hexagonal QC with piezoelectric effect [20]. Zhang et al. [21] analyzed a plane problem of 1D orthorhombic QC with piezoelectric effect. As we know, a QC in practical use has finite size, and consideration of the effect of boundaries is necessary.

This paper considers a hexagonal piezoelectric QC of finite thickness in the periodic plane with two collinear cracks. The objective is to seek the closed-form solution of electroelastic field induced by two collinear cracks when the QC is subjected to antiplane shear loading. Using the Fourier transform and triple integral equations, we reduce the problem to a singular integral equation. The desired solution is then derived in terms of the complete elliptic integral of the first and the third kinds. The closed-form solutions of phonon and phason elastic fields and the electric fields are obtained. Fields intensity factors and energy release rates are given and displayed graphically.

2. Statement of the problem

2.1. Basic equations for piezoelectric QC

Consider a 1D hexagonal piezoelectric QC with point group 6 mm. Introduce Cartesian coordinates (x, y, z) such that the quasiperiodic poling axis is chosen as the z-axis, and an isotropic periodic plane perpendicular to the quasicrystal poling axis