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Evaluation of stress intensity factors under multiaxial and compressive conditions using low order displacement or stress field fitting

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ABSTRACT

A methodology for evaluation of stress intensity factors from the asymptotic displacement (or stress) fields at a crack front is proposed. The methodology strives to be of practical use in conjunction with commercial FE-codes, also when approaches such as XFEM are employed. To this end, the matching of the fields is carried out through a minimalistic, low order, ansatz of the displacement and stress fields. As demonstrated, the proposed methodology can deal with multiaxial loading of curved crack fronts in three-dimensional bodies. The methodology shows good accuracy also for compressive loading when crack face friction can be neglected.

1. Introduction

The safety and reliability of structures and load bearing components often depend on whether cracks are present and, if so, whether they will propagate or not. In this context fracture mechanics approaches are of great importance and widely used within e.g. the nuclear industry, aeronautics, ship building and railway engineering. Ever since Irwin [1] introduced the concept of Stress Intensity Factors (SIFs) these have been valuable engineering tools for quantifying the severity of cracks. For some geometries and load cases the SIFs can be evaluated from handbook solutions, see e.g. Tada et al. [2]. However, closed-form solutions are generally not available for three dimensional cracks of arbitrary shape and loading conditions. For such structures numerical methods must be relied upon.

There is a wealth of literature devoted to numerical evaluation of SIFs, see e.g. Sih [3], Atluri [4] and Kuna [5]. A common approach is to use the Finite Element Method (FEM) to evaluate the displacement and stress fields of a cracked body. The SIFs can then be evaluated directly, either by extrapolating stresses towards the crack front in the Stress Interpretation Method (SIM) or via asymptotic relations of the displacements close to the crack front in the Displacement Interpretation Method (DIM) [5]. These methods are robust but require a dense mesh close to the crack tip. Another, perhaps more common, evaluation technique, available in several commercial FE codes, is to evaluate SIFs from the J-integral (cf. Rice [6]) or its generalised counterparts such as the domain integral (cf. Shih et al. [7]). Since the need to accurately resolve the stress singularity close to the crack tip is reduced in this method, a coarser mesh can be used. Another computationally efficient technique for

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Nomenclature

Acronyms

CAE	Computer-Aided Engineering
DIM	Displacement Interpretation Method
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method
LEFM	Linear Elastic Fracture Mechanics
RCF	Rolling Contact Fatigue
SIF	Stress Intensity Factor
SIM	Stress Interpretation Method
XFEM	eXtended Finite Element Method

Symbols

x, y, z	Cartesian coordinates
r, θ	polar coordinates
E	Young's modulus
ν	Poisson's ratio
u_i	displacement vector
σ_{ij}	stress tensor
σ	normal stress
τ	shear stress
a	crack length
$K_{\mathrm{I}},\ K_{\mathrm{II}},\ K_{\mathrm{III}}$	stress intensity factors
$K_{\mathrm{I,t}}, K_{\mathrm{II,t}},\ K_{\mathrm{III,t}}$	stress intensity factors obtained from handbook solutions
h	characteristic element length
V	integration domain
$d_x,\ d_y,\ d_z$	side lengths of integration domain
$\bar{r}_{\mathrm{l}}$	lower cut off-distance inside integration domain
$\bar{r}_{\mathrm{u}}$	upper cut off-distance inside integration domain
w	weight function inside integration domain
$\mathbf{u}^{\mathrm{a}}$	displacement ansatz
$\mathbf{u}^{\mathrm{FE}}$	finite element displacements
$\mathbf{A}_{\mathrm{K}},\ \mathbf{A}_{\mathrm{RBM}},\ \mathbf{A}_{\bar{\epsilon}}$	displacement ansatz basis functions representing K-fields, rigid body motion and uniform strain
$\mathbf{b}_{\mathrm{K}},\ \mathbf{b}_{\mathrm{RBM}},\ \mathbf{b}_{\bar{\epsilon}}$	displacement ansatz coefficients
$\mathbf{A}$	combined displacement ansatz basis functions
$\mathbf{b}$	combined displacement ansatz coefficients
$\mathbf{s}^{\mathrm{a}}$	stress ansatz
$\mathbf{s}^{\mathrm{FE}}$	finite element stresses
$\mathbf{A}_{\sigma,\mathrm{K}},\ \mathbf{A}_{\sigma,1},\ \mathbf{A}_{\sigma,r}$	stress ansatz basis functions representing K-fields, constant terms and linear terms
$\mathbf{b}_{\sigma,\mathrm{K}},\ \mathbf{b}_{\sigma,1},\ \mathbf{b}_{\sigma,r}$	stress ansatz coefficients
$\mathbf{A}_{\sigma}$	combined stress ansatz basis functions
$\mathbf{b}_{\sigma}$	combined stress ansatz coefficients

SIF evaluation is to use so-called hybrid crack tip elements, where the SIFs are solved for as primary unknowns in contrast to being obtained from a post-processing step. This type of element is however seldom available in commercial software [5].

If the SIM, DIM or the J-integral approach is to be used with conventional FEM, it is required that the crack is accounted for during the meshing procedure. In the J-integral case special techniques must be used to mesh the region around the crack front, see e.g. [8]. One drawback with this procedure is the high meshing effort needed if several crack geometries are to be analysed, e.g. in a parametric study or if the crack is allowed to propagate. This issue could be resolved by the introduction of the eXtended Finite Element Method (XFEM), cf. Belytschko and Black [9]. Here, the crack is modelled by enhancing the conventional FE approximation by additional shape functions and unknowns. In that manner a crack can be modelled as a strong discontinuity in the displacement field and special functions may (or may not) be added to the crack front elements in order to resemble known behaviour close to the crack tip. In this way there is no need to actually mesh the crack geometries. These are instead modelled implicitly via so called level set functions that can easily be modified without altering the mesh. XFEM