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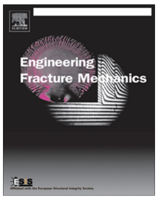

Discrete elements and size effects

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A B S T R A C T

The discrete element method (DEM) in use today generally considers the bonds that bind discrete elements into a continuum to be elasto-perfectly-brittle. Such brittle bonds only give accurate failure loads for large size structures where LEFM applies. But for small to medium size structures of quasi-brittle materials such as rocks and concretes, the presence of the fracture process zone (FPZ) has significant impacts and needs to be considered. To properly model size effects, an exponential softening contact bonds model was implemented which is tied to input fracture energy and also addresses particle size issue. This was followed by a successful study of the classical size effect problems of similar edge notched beams subjected to direct tension and three-point bending. The overestimation of failure loads by brittle bonds are then addressed. The discrete nature of DEM also facilitates the use of different particle packing for structure construction, and provided insights on how FPZ evolves under sustained loading.

1. Introduction

Because of the nature of its distinctive construct, the discrete element method [1–3] has been widely used in failure analysis for quasi-brittle materials such as rocks and concretes [4–8]. In a discrete element modeling, a continuum is represented as an assembly of particles or elements bonded together. When bonds between two particles failed, they are removed and fractures are introduced. No special consideration is required to model fracture progression as a failure cascades. Furthermore, the mechanical interactions across newly formed fractured surfaces are automatically facilitated. This makes DEM an appealing methodology for studying failures [4–8] and fractures [9–12]. However, most of the DEM in use today considers elasto-perfectly-brittle bonds [2,13–15], referred to as brittle bonds hereafter, and such bonds are good for modeling fracture problems that follow the theory of LEFM but not for modeling size effects of quasi-brittle materials.

It has been well established that fracture and crack growth in quasi-brittle materials, such as rocks and concretes, are associated with a fracture process zone (FPZ) ahead of the crack tip. This zone, containing micro-cracks, is responsible for strain-softening behavior of structures under loading, and incorporating micro-cracks into modeling is a key to capture the fracture process and related fracture characteristics such as size effect.

Efforts have been made to address this issue by the introduction of strain softening bonds [15–17]. On the application side, Kim et al. [18–21] employed an intrinsic linear softening model in PFC^{2D} to study the fracture behavior of asphalt

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Nomenclature

a, a_0	extended and initial crack length
B, D_0	dimensionless geometric factor and critical transition size of size effect
c	cohesion of bond
c_n	a configuration dependent constant for nominal stress
C_r	a reduction factor for calibrating fracture energy
D, L	depth and length of beam
D_f	damage factor
E'	effective Young's modulus
F, F_n, F_s	contact force, normal contact force and shear contact force
F_u	peak load
G_f	fracture energy
k, k_0	dimensionless shape factor for the stress intensity factor
K, K_n, K_s	stiffness of contact
K_I, K_II	stress intensity factor
K_IC, K_IIC	fracture toughness
l_ch	characteristic length
L_c	length of crack along depth direction
L_p, L_pmax	length and maximum length of fracture process zone
n	power of parabolic stress distribution
n_i	unit-normal vector directed from a particle centroid to its contact location
N_c	number of contacts for each particle that lies within measurement circle
N_d	total number of particles with centroids contained within measurement circle
N_p	particle number within fracture process zone
r	a parameter of size effect law
r_eq	equivalent distance of particle to crack tip
R, R_max, R_min	radius of disk
$\bar{R}$	average radius of disk
R_m	radius of measurement circle
S_n, S_s	initial normal and shear strength of contact
$S_\mathrm{n,sof}$, $S_\mathrm{s,sof}$	softened normal and shear strength of contact
$S'_\mathrm{s,sof}$	softened shear strength of contact under compression
t	thickness of disk
u, u_n, u_s	displacement of contact
u_e	limit elastic displacement of contact
u_f, u_f^n, u_f^s	softening parameter
$V^{(\mathrm{d})}$	the volume of particle
α, α_0	relative crack length
α', β'	two correction factors
θ	polar coordinate of crack tip
μ	friction coefficient
σ_f	far field tensile stress
$\bar{\sigma}_{ij}$	average stress tensor in a measurement circle
σ_n, σ_n^i	stress normal to the crack plane
σ_N,σ_{Nu}	nominal and peak nominal stress
σ_t	tensile strength
ϕ	porosity within the measurement region

concrete under compact tension test but no fracture properties related to quasi brittle materials were studied. Fakhimi et al. [16,22–24] employed a linear softening constitutive for normal spring and kept shear spring as brittle bond to simulate tensile and mixed-mode fracture in sandstone. Because only few works have so far been carried out, a systematic study on how the introduction of softening bonds could improve modeling is still lacking. This leads to ambiguity about how well and how much detail the DEM results can shed lights on the fracture process. Moreover, it is also not clear that how the use of brittle bonds might lead to overestimates of the failure loads for small to medium size structures. To address these issues, this study starts from the basic construct of DEM, works through fracture formulations, and carries out a comprehensive analysis.

In comparison with DEM, the continuum approach of modeling fracture and the fracture process is rather mature: it has led to important understanding, and provided good estimates of fracture process zone [25–27] as well as size effect [28–30]. In a continuum approach, micro-cracks are often explicitly modeled with cohesive model or implicitly modeled in which the