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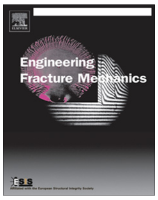

Subcycle fatigue crack growth formulation under positive and negative stress ratios

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ABSTRACT

A new fatigue crack growth model at the subcycle scale is proposed for both positive and negative stress ratios. The term "subcycle" refers to the crack growth modeling within a cyclic loading duration rather than the classical cycle averaged crack growth rate (i.e., da/dN). The proposed model includes two major components: a subcycle crack growth rate function and a crack opening stress estimation algorithm for tension–tension and tension–compression cyclic loading. The subcycle crack growth rate function is proposed based on existing in situ scanning electron microscopy testing results. Following this, separation of far field crack contact and near tip crack contact are discussed for both positive and negative stress ratios. A simple linearized model is proposed to estimate the crack opening stress. The developed simplified crack opening stress estimation model is compared with several other models available from open literatures. Next, the proposed subcycle fatigue crack growth methodology is validated with several sets experimental data for various metallic materials under different positive and negative stress ratios. Some discussions and conclusions are drawn based on the proposed model.

1. Introduction

Fatigue crack growth (FCG) has been extensively studied in the open literature for life prediction and design of metallic materials and structures. One of the most commonly used approach is to correlate the fatigue crack growth propagation rate per cycle da/dN with the stress intensity factor (SIF) range ΔK as

$$\frac{da}{dN} = C\Delta K^m \tag{1}$$

where C and m are fitting parameters dependent on materials and stress ratios. This is the well-known Paris' law [1]. One of known limitations of the Paris' law is that it is incapable to handling the stress ratio effects and many modifications have been proposed [2]. Among these approaches, the crack closure concept originally proposed by Elber [3] is one of the widely used mechanical models to explain the stress ratio effect. The key idea is that an effective stress intensity range where the crack remains is responsible for the stress ratio effect on the fatigue crack growth (FCG) rate. It is shown that [3] a crack would open and propagate at the opening portion of the cycling load, which can be defined as

$$\Delta K_{eff} = K_{max} - K_{op} \tag{2}$$

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Nomenclature

da/dN	fatigue crack growth rate per cycle
C	Paris law coefficient
m	Paris law exponent
ΔK	stress intensity factor range (SIF)
ΔK_{eff}	effective SIF
ΔK_{eq}	equivalent SIF
K_{max}	maximum stress intensity SIF
K_{op}	crack opening SIF
R	cyclic stress ratio
a	crack length at any arbitrary instant of time
Δa	incremental crack growth in the one cycle
da	infinitesimal crack increment
A, B	subcycle model material parameters
δ	crack tip opening displacement
S_{max}, σ_{max}	maximum stress in one loading cycle
S_{min}, σ_{min}	minimum stress in one loading cycle
S_{lc}, S_{gc}, σ_{lc}, σ_{gc}	local crack contact and global crack contact stress in one cycle
K_{op}, σ_{op}	crack opening stress level
K_{cl}, σ_{cl}	crack closure stress level
d_r	reversed plastic zone size at any instant
d	forward plastic zone size
β	dimensionless parameter for global crack opening stress
σ_y	material yield strength
E	Young's modulus

where $\Delta K_{eff}, K_{max}, K_{op}$ are the effective, maximum and crack opening SIF, respectively.

Many other studies have been proposed using the crack closure concept to explain the stress ratio effect in the FCG analysis. In the literature, different empirical models for crack closure have been proposed for both positive and negative ratios. In all these models, the FCG and crack opening behavior are very different under positive stress ratio and negative stress ratios. It had been observed that the FCG is strongly affected by the local plastic deformation at the crack tip region [4]. Several literature results based on tests performed under tension–compression fatigue loading indicates that the effect of compressive load on the FCG was strongly material dependent [5]. Several empirical models based on finite element analysis have been proposed for estimating crack opening stress under negative stress ratios [6–9]. A brief summary for the literature results on the crack opening stress estimation is shown in Table 1. It is observed that each model has their own applicability ranges with respect to the stress ratio range.

Most existing models on FCG analysis is based on the cycle definition of crack growth rate (e.g., da/dN) and driving force parameter ranges (e.g., ΔK). Very few studies investigate the FCG behavior using the subcycle formulation within a cyclic loading. Here, the term "subcycle" refers to the formulation within a cycle. Lu and Liu [10] proposed the subcycle formulation for the investigation of FCG behavior of materials by using several hypothesis to define the subcycle FCG rate function within a cycle. Later on, Zhang and Liu [11,12] performed in situ scanning electron microscopy (SEM) testing to directly observe the subcycle FCG behavior. Several unique benefits for the subcycle formulation are that they can explicitly include the subcycle fatigue crack growth mechanisms, which are observed to be different at different portions within a cycle [11,12]. In addition, no cycle counting is required for the subcycle formulation and direct crack growth calculation with the time varying stress history can be achieved by using the direct integration [13]. A simplified crack opening stress estimation is proposed based on the in situ SEM observations [12], but the model is only applicable to positive stress ratios and is only validated with Al-7075-T6. Thus, one of the major objectives of the current study is to extend the formulation to both positive and negative stress ratios and to check the applicability to various other metallic materials.

Table 1
Literature models for crack opening stress level based on stress ratio *R*.

Author	Model	Material	Model validity
Elber [3]	$\sigma_{op}/\sigma_{max} = 0.5 + 0.1R + 0.4R^2$	2024-T4	$-0.1 < R < 0.7$
Schijve [8]	$\sigma_{op}/\sigma_{max} = 0.45 + 0.22R + 0.21R^2 + 0.12R^3$	2024-T4	$-1 < R < 0.5$
Zhang [5]	$\sigma_{op}/\sigma_{max} = 1 - (1 - R)(0.618 + 0.365R + 0.139R^2)$	7475-T73	$-1 < R < 0.5$
Kumar [9]	$\sigma_{op}/\sigma_{max} = 1 - (1 - R)(0.7 + 0.15R(2 + R))$	Steel	$-1 < R < 0.5$
Lang [7]	$\sigma_{op}/\sigma_{max} = 0.453 + 0.34R + 0.134R^2 + 0.07R^3$	7475-T7351	$-0.7 < R < 1$
Newman [6]	$\sigma_{op}/\sigma_{max} = 0.535 + 0.069R + 0.139R^2 + 0.257R^3$	7075-T6	$-1 < R < 0.9$