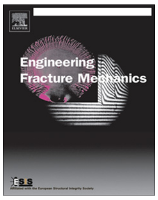
Engineering Fracture Mechanics xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech



A simplified evaluation of the mechanical energy release rate of kinked cracks in piezoelectric materials using the boundary element method

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ARTICLE INFO

Article history:
Received 23 May 2017
Received in revised form 7 July 2017
Accepted 7 July 2017
Available online xxxx

Keywords:
Crack problems
Mechanical energy release rate
Fracture criteria
Piezoelectric materials
Traction BIEs

ABSTRACT

Based on the concept of the hoop field intensity factors of an initial crack prior to any kink, an apparent hoop mechanical (strain) energy release rate (MERR) is defined to approximate the MERR of a piezoelectric crack with an infinitesimal kink at any arbitrary angle. The validity and the efficiency of the simplified approximation are examined by numerical examples using the boundary element method (BEM). The generalized crack-opening-displacements or displacement jumps are computed by the traction boundary integral equations (BIEs). By using the displacement extrapolation method, the crack-tip field intensity factors of any arbitrarily kinked crack in linear piezoelectric materials are obtained and the BEM results are validated by comparing them with the available reference analytical results. Then, the differences between the conventional field intensity factors and MERR of an infinitesimally kinked crack and the hoop field intensity factors and hoop MERR of the main crack prior to any kink are numerically analyzed. Finally, the crack propagation in an infinite linear piezoelectric material is numerically simulated. The paths of the crack growth are predicted by adopting four different fracture criteria, namely, the maximum hoop stress intensity factor (SIF) and MERR fracture criteria for the main crack-tip before the next propagation, and the maximum K_I and MERR fracture criteria for the kinked tip of the main crack with an infinitesimal branch at an arbitrary kinking angle evaluated by using a trial crack extension technique. The comparisons among these results show that the present simplified approximation can efficiently provide a sufficient accuracy for numerical simulation of crack growth in linear piezoelectric materials.

1. Introduction

Due to the electro-mechanical coupling behavior, piezoelectric ceramics (such as PZT-4, PZT-5h, PZT-6b, and $BaTiO_3$) are widely used in smart structures and electronic devices. Unfortunately, commercial available piezo-ceramics are rather susceptible to fracture and damage due to their inherent brittleness and low fracture toughness, especially under highly concentrated stresses and electrical fields. To assure a sufficient reliability of technical devices for advanced engineering

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http://dx.doi.org/10.1016/j.engfracmech.2017.07.008
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Please cite this article in press as: Lei J, Zhang C. A simplified evaluation of the mechanical energy release rate of kinked cracks in piezoelectric materials using the boundary element method. Engng Fract Mech (2017), http://dx.doi.org/10.1016/j.engfracmech.2017.07.008

Nomenclature

Latin symbols

A	characteristic crack-length
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_1$	material matrices related to the characteristic roots and eigenvectors
$\mathbf{a}_J$	eigenvectors of the characteristic equation
$\mathbf{b}_J$	vectors defined as $(\mathbf{R}^T + \mu_J \mathbf{T})\mathbf{a}_J$
c_{ijkl}	elasticity tensor
c_{iJKl}	extended elasticity tensor
c_{ij}	elasticity matrix
$\mathrm{D_i}$	electric displacement vector
D_ω	hoop electric displacement vector
$\mathrm{E_i}$	electric field vector
e_{kij}	third-order piezoelectric tensor
e_{kq}	piezoelectric matrix
$\boldsymbol{f}(z)$ or $f(z_i)$	complex analytic functions
G	total crack-tip ERR
G_{M}	MERR
G_ω^{m}	hoop ERR
$G_{\mathrm{M}\omega}^{\mathrm{m}}$	hoop MERR
$G_\omega^{\mathrm{Irwin}}$	ERR of a small crack-kink evaluated by the modified Irwin formula
$G_{\mathrm{M}\omega}^{\mathrm{Irwin}}$	MERR of a small crack-kink evaluated by the modified Irwin formula
$\mathbf{K}(K_I, K_{II}, K_D)$	generalized SIFs
K_I or K_1	mode-I SIF
K_{II} or K_2	mode-II SIF
K_D or K_3	electric displacement intensity factor
$\mathbf{K}^{\mathrm{b}}$	generalized SIFs of an infinitesimally small crack-kink
$\mathbf{K}_\omega(K_{\omega\omega}, K_{r\omega}, K_{D\omega})$	generalized hoop SIFs
$K_{\omega\omega}$	hoop SIF
$K_{r\omega}$	shear SIF
$K_{D\omega}$	hoop electric displacement intensity factor
$\mathbf{K}_\omega^{\mathrm{m}}$	hoop intensity factors of the main crack
$\mathbf{L}$ or L_{ij}	material matrix defined as $2\mathrm{Re}[\mathbf{Y}]$
L	length of the small kinked branch
n_i	outward unit normal vector
p_J	traction/surface charge vector
r	polar coordinate
s_{IJ}^*	traction fundamental solutions
$\mathbf{t}_1$	traction vector defined as $\{\sigma_{11}, \sigma_{12}, D_1\}^{\mathrm{T}}$
$\mathbf{t}_2$	traction vector defined as $\{\sigma_{21}, \sigma_{22}, D_2\}^{\mathrm{T}}$
$\mathbf{u}$	extended displacement vector
u_i	elastic displacement vector
$\Delta\mathbf{u}$	generalized crack-opening-displacements
$\mathbf{Q}$, $\mathbf{R}$, $\mathbf{T}$	system matrices of the characteristic equation
$\mathbf{x}$	observation point
$\mathbf{Y}$	Irwin matrix defined as $i\mathbf{AB}^{1}$
z_M, z_M^0	counterpart of the source and the observation points

Greek symbols

ξ	source point
δ	distance to the crack-tip
$\boldsymbol{\varepsilon}$	extended strain tensor
ε_{ij}	mechanical strain tensor
κ_{ij}	second-order dielectric tensor
ϕ	electrical potential
ϕ_M	shape functions
$\Re$	coordinate transformation matrix
μ_J	complex roots of the characteristic equation

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