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## A novel conjugated bond linear elastic model in bond-based peridynamics for fracture problems under dynamic loads

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#### ABSTRACT

A novel conjugated bond linear elastic model is proposed and implemented into the bondbased peridynamic (BB-PD) framework. In this model, micro-elastic PD bond energy is not only related to the normal stretch of bonds, but also related to the rotation bond angles of a pair of conjugated bonds. Therefore, micro bond energy mechanism in this study is different from that in the classical continuum mechanism or the standard BB-PD. Only one micro-elastic constant in the standard BB-PD results in the limitation of the effective Poisson's ratio for isotropic material. However, the novel conjugated bond linear elastic model incorporating two micro-elastic constants are proposed, which can overcome the limitation of Poisson's ratio in the standard BB-PD. By comparing the strain energy in the proposed model with that in the classical elastic model, the corresponding micromacro parameter relationships can be established. In addition, energy-based bond rupture criteria are implemented in the proposed numerical model to simulate fracture problems under dynamic loads. In order to verify the ability and accuracy of the proposed numerical model to simulate fracture problems under dynamic loads, some numerical examples are investigated. The present numerical results are in good agreement with the previous experimental and numerical results.

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#### 1. Introduction

In computation mechanics, numerical simulation of fracture problems for both static cracks and dynamic cracks is still an active and persistent challenge. There are some different methods to model cracks in finite element and meshfree methods. For the finite element method, one of the simple and robust methods is interelement separation model where cracks are modeled along element interfaces in the mesh [1–4]. Another simple method to model cracks was developed by Remmers et al. [5] who introduced crack segments in finite elements. The embedded discontinuity model was also proposed by Armero and Garikipati [6], Belytschko et al. [7], Samaniego et al. [8] in the finite element mesh to simulate crack problems. Based on the 'local' partition of unity, the extended finite element method (XFEM), which is a very accurate method for crack problems, was developed by Belytschko and Black [9] and Moes et al. [10]. For the previous meshfree method, Rabczuk and Belytschko [11,12] developed a 'cracking particle' model (CPM) in meshfree method, where discontinuities are introduced at

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#### X. Zhou et al./Engineering Fracture Mechanics xxx (2017) xxx-xxx

$C(\xi)$ micro-modulus constant in bond-based peridynamics $C_n$ normal stiffness of bonds $D$ ratio of amount of broken bonds to the total amount of interaction in one horizon $E$ Young's modulus $E$ classic Lagrange strain tensor $f$ pairwise force function in conjugated bond-based peridynamics $f_n$ normal bond force density in conjugated bond-based peridynamics $f_n$ angent bond force density $F$ approximate deformation gradient $G_s$ fracture surface energy $H_X$ horizon of a given material point XIidentity tensor $K$ bulk elastic modulus $m_{RW}$ moment vector density due to a pair of conjugated bonds $s$ bond stretch $m_{RW}$ moment vector density due to a pair of conjugated bonds $s$ bond stretch $w$ scalar-valued micro-potential function $w_i$ caceleration vectors of the material point X $w_i$ caceleration vectors of the material point X $w_i$ mortain energy micro-potential $w_i$ cacitar-valued micro-potential function $w_i$ micro-potential function $w_i$ micro-potential energy function $w_i$ micro-potential energy density $w_i$ critical strain energy density $w_i$ material point X in energy density $w_i$ material energy density $w_i$ material energy density $w_i$ material energy density $w_i$ micro-potential energy density	Nomenclature	
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$W^{PD}$ macro-elastic strain energy density $W^{PD}_{\beta}$ macro-elastic stretch strain energy density $W^{PD}_{\beta}$ macro-elastic rotation strain energy density $\mathbf{x}$ location of a material point in deformed configuration $\mathbf{x}'$ neighbor particle for a given point $\mathbf{x}$ in deformed configuration $\mathbf{x}'$ neighbor particle for a given point $\mathbf{x}$ in the reference configuration $\mathbf{X}'$ neighbor particle for a given point $\mathbf{X}$ in the reference configuration $\mathbf{X}'$ neighbor particle for a given point $\mathbf{X}$ in the reference configuration $\delta$ radius of one horizon $\varepsilon_{ij}$ strain tensor in the bond $\xi_{ij}$ $\theta_{jiko}$ conjugated bond angle in the reference configuration $\theta_{jik}$ conjugated bond angle in the deformed configuration $\Theta_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\lambda_t$ rotation resistance stiffness $\mu$ scalar factor representing the broken of bond $\nu$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi'$ unit orientation vector of the conjugated bond	$W^{c}$	strain energy density in classical elastic mechanics
$ \begin{array}{ll} W_{\beta}^{1D} & \text{macro-elastic stretch strain energy density} \\ W_{\beta}^{DD} & \text{macro-elastic rotation strain energy density} \\ \textbf{x} & \text{location of a material point in the formed configuration} \\ \textbf{x}' & \text{neighbor particle for a given point } \textbf{x} & \text{in deformed configuration} \\ \textbf{X} & \text{location of a material point in the reference configuration} \\ \textbf{X} & \text{neighbor particle for a given point } \textbf{X} & \text{in the reference configuration} \\ \textbf{X} & \text{neighbor particle for a given point } \textbf{X} & \text{in the reference configuration} \\ \textbf{X} & \text{neighbor particle for a given point } \textbf{X} & \text{in the reference configuration} \\ \textbf{X} & \text{neighbor particle for a given point } \textbf{X} & \text{in the reference configuration} \\ \textbf{X} & \text{neighbor particle for a given point } \textbf{X} & \text{in the reference configuration} \\ \textbf{X} & \text{neighbor particle bond angle in the reference configuration} \\ \textbf{X} & \text{conjugated bond angle in the reference configuration} \\ \textbf{X} & \text{conjugated bond angle in the deformed configuration} \\ \textbf{Y} & \text{rotation angle of conjugated bonds in the deformed configuration} \\ \textbf{Y} & \text{rotation resistance stiffness} \\ \mu & \text{scalar factor representing the broken of bond} \\ \textbf{V} & \text{Poisson's ratio} \\ \textbf{\xi} & \text{bond vector in the reference configuration} \\ \textbf{Y} & \text{relative displacement vector between two interacting material points} \\ \textbf{\rho} & \text{material density} \\ \textbf{Y} & \text{unit orientation vector of the bond} \\ \boldsymbol{\psi} & \text{unit orientation vector of the conjugated bond} \\ \end{array}$	$W^{PD}$	macro-elastic strain energy density
$W_{jb}^{o}$ macro-elastic rotation strain energy density $\mathbf{x}$ location of a material point in deformed configuration $\mathbf{x}'$ neighbor particle for a given point $\mathbf{x}$ in deformed configuration $\mathbf{X}'$ location of a material point in the reference configuration $\mathbf{X}'$ neighbor particle for a given point $\mathbf{X}$ in the reference configuration $\delta'$ radius of one horizon $\varepsilon_{ij}$ strain tensor in the bond $\xi_{ij}$ $\theta_{jiko}$ conjugated bond angle in the reference configuration $\theta_{jik}$ conjugated bond angle in the deformed configuration $\Theta_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\lambda_t$ rotation resistance stiffness $\mu$ scalar factor representing the broken of bond $v$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	$W_l^{PD}$	macro-elastic stretch strain energy density
$\mathbf{x}$ Indication of a material point in deformed configuration $\mathbf{x}'$ neighbor particle for a given point $\mathbf{x}$ in deformed configuration $\mathbf{X}'$ location of a material point in the reference configuration $\mathbf{X}'$ neighbor particle for a given point $\mathbf{X}$ in the reference configuration $\delta$ radius of one horizon $\varepsilon_{ij}$ strain tensor in the bond $\xi_{ij}$ $\theta_{jik0}$ conjugated bond angle in the reference configuration $\theta_{jik0}$ conjugated bond angle in the deformed configuration $\Theta_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\lambda_t$ rotation resistance stiffness $\mu$ scalar factor representing the broken of bond $\nu$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	$W^{ID}_{\beta}$	macro-elastic rotation strain energy density
XIntegration particle for a given point X in deformed configurationXlocation of a material point in the reference configuration $X'$ neighbor particle for a given point X in the reference configuration $\delta$ radius of one horizon $\varepsilon_{ij}$ strain tensor in the bond $\xi_{ij}$ $\theta_{jik0}$ conjugated bond angle in the reference configuration $\theta_{jik}$ conjugated bond angle in the deformed configuration $\Theta_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\phi_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\psi$ scalar factor representing the broken of bond $v$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	x x'	nocation of a material point in deformed configuration
$\mathbf{X}'$ neighbor particle for a given point $\mathbf{X}$ in the reference configuration $\delta$ radius of one horizon $\varepsilon_{ij}$ strain tensor in the bond $\xi_{ij}$ $\theta_{jik}$ conjugated bond angle in the reference configuration $\theta_{jik}$ conjugated bond angle in the deformed configuration $\Theta_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\phi_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\psi$ rotation resistance stiffness $\mu$ scalar factor representing the broken of bond $v$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	x X	location of a material point in the reference configuration
$\begin{array}{llllllllllllllllllllllllllllllllllll$	x′	neighbor narticle for a given noint $\mathbf{X}$ in the reference configuration
$\varepsilon_{ij}$ strain tensor in the bond $\xi_{ij}$ $\theta_{jik0}$ conjugated bond angle in the reference configuration $\theta_{jik}$ conjugated bond angle in the deformed configuration $\theta_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\Delta_t$ rotation resistance stiffness $\mu$ scalar factor representing the broken of bond $\nu$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	δ	radius of one horizon
$ \begin{array}{ll} \theta_{jik0} & \mbox{conjugated bond angle in the reference configuration} \\ \theta_{jik} & \mbox{conjugated bond angle in the deformed configuration} \\ \theta_{jik} & \mbox{rotation angle of conjugated bonds in the deformed configuration} \\ \lambda_t & \mbox{rotation resistance stiffness} \\ \mu & \mbox{scalar factor representing the broken of bond} \\ \nu & \mbox{Poisson's ratio} \\ \xi & \mbox{bond vector in the reference configuration} \\ \eta & \mbox{relative displacement vector between two interacting material points} \\ \rho & \mbox{material density} \\ \chi & \mbox{unit orientation vector of the bond} \\ \psi & \mbox{unit orientation vector of the conjugated bond} \end{array} $	Eii	strain tensor in the bond $\xi_{ii}$
$\theta_{jik}$ conjugated bond angle in the deformed configuration $\theta_{jik}$ rotation angle of conjugated bonds in the deformed configuration $\lambda_t$ rotation resistance stiffness $\mu$ scalar factor representing the broken of bond $v$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	$\theta_{iik0}$	conjugated bond angle in the reference configuration
	$\theta_{jik}$	conjugated bond angle in the deformed configuration
$\lambda_t$ rotation resistance stiffness $\mu$ scalar factor representing the broken of bond $v$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	$\hat{\Theta}_{jik}$	rotation angle of conjugated bonds in the deformed configuration
$\mu$ scalar factor representing the broken of bond $v$ Poisson's ratio $\xi$ bond vector in the reference configuration $\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	$\lambda_t$	rotation resistance stiffness
<ul> <li>ν Poisson's ratio</li> <li>ξ bond vector in the reference configuration</li> <li>η relative displacement vector between two interacting material points</li> <li>ρ material density</li> <li>χ unit orientation vector of the bond</li> <li>ψ unit orientation vector of the conjugated bond</li> </ul>	$\mu$	scalar factor representing the broken of bond
<ul> <li><i>ξ</i> bond vector in the reference configuration</li> <li><i>η</i> relative displacement vector between two interacting material points</li> <li><i>ρ</i> material density</li> <li><i>χ</i> unit orientation vector of the bond</li> <li><i>ψ</i> unit orientation vector of the conjugated bond</li> </ul>	v	Poisson's ratio
$\eta$ relative displacement vector between two interacting material points $\rho$ material density $\chi$ unit orientation vector of the bond $\psi$ unit orientation vector of the conjugated bond	ξ	bond vector in the reference configuration
$\begin{array}{ll} \rho & \text{inaterial density} \\ \chi & \text{unit orientation vector of the bond} \\ \psi & \text{unit orientation vector of the conjugated bond} \end{array}$	η	relative displacement vector between two interacting material points
$\psi$ unit orientation vector of the conjugated bond	$\rho$	Induction definition vector of the bond
	K	unit orientation vector of the conjugated bond
	Ψ	

the particle positions. The visibility criterion or some modification of it were also proposed in the meshfree methods for cracking [13–16]. Ventura et al. [17] enriched the MLS base functions around the crack tip to model kinked and curve cracks.

Although these numerical methods have been successfully applied to simulate the fracture problems, theories of these methods are in the framework of continuum mechanics, whose governing equations of motion are based on partial derivatives with respect to the spatial coordinates. The assumption of continuity in continuum mechanics is inherently insufficient for modeling cracks as partial derivatives are undefined along the crack faces where the displacement field is discontinuous. Therefore, some computation methods involving displacement gradients or higher-order spatial derivatives in a domain containing a crack must remove the discontinuous displacement field by redefining the discretized body so that the crack lies on the boundary or by using other techniques for evaluating the spatial derivatives on crack surfaces [9,18,19]. However, the peridynamic formulation eliminates the spatial derivatives altogether by solely depending on an integral formulation of the force acting at a continuum point, resulting in equilibrium equations that are valid everywhere in the body [20].

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2

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