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A new symplectic analytical singular element for crack problems under dynamic loading condition

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ABSTRACT

In this paper, the two-dimensional (2D) crack problem under dynamic loading condition is studied. Firstly, by the precise time domain expanding algorithm, the recursive formulation of the governing equation is derived, and the recursive equation can be solved by FEM. Then, a new Symplectic Analytical Singular Element (SASE) defined by the analytical symplectic eigen expanding terms is further developed for the investigated problem. The area near the crack tip is represented by using the SASE, while the other area is meshed by conventional elements. Taking advantage of the SASE, the crack problem can be solved with higher accuracy. Simultaneously, the dynamic stress intensity factors (DSIFs) can be obtained directly without any post processing through the relationship between the DSIFs and the eigen expanding coefficients. The numerical accuracy and stability is investigated carefully through the conducted numerical examples. It has been demonstrated that the proposed SASE for dynamic crack problem is effective and efficient.

1. Introduction

Dynamic stress intensity factors (DSIFs) are of particular interest in characterizing and predicting fracture behavior of cracked structures subjected to dynamic loadings. Several methods have been developed and applied by researchers to evaluate DSIFs for various problems in the frame of linear elastic fracture mechanics (LEFM), including the finite difference method (FDM) [1], the element-free Galerkin (EFG) method [2], finite element method (FEM) [3,4], boundary element method (BEM) [5,6], etc. Among them, FEM has become the most popular method in recent years.

Kishimoto et al. [3] determined DSIFs using the formula derived from a modified path independent J-integral, and the problem of a suddenly loaded crack was analyzed using the conventional FEM. Nishioka and Atluri [7,8] investigated rapidly propagating crack problem (mode I) with known propagation speed in 2D finite bodies and proposed an rectangular singular element method. The asymptotic solutions for the dynamic crack problem are used to construct the singular element. The formulation of the singular element is related to the type of the surrounding regular element and cannot be determined unless the type of the surrounding elements is chosen. Murti and Valliappan [9] investigated the transient dynamic problem, using the quarter-point elements (QPEs) in combination with FEM. DSIFs were evaluated by using the relationship between the displacement in the vicinity of the crack and the first two coefficients of Williams solution [10]. Enderlein et al. [11] investigated the fracture behavior of 2D and 3D cracked bodies subjected to impact loading using FEM. The pure mode I DSIF

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were evaluated by utilizing different methods, including the dynamic J-integral, the modified crack closure integral and the displacement correlation technique. Saribaya and Nied [12] used the enriched crack tip element in conjunction with FEM to determine DSIFs. However, transition elements are required to maintain displacement compatibility between the enriched crack-tip elements and regular elements. The scaled boundary finite element method (SBFEM) developed by Song and Wolf [13] has been applied to fracture problems. Ooi et al. [14,15] dealt with dynamic fracture problem using the SBFEM. Chen et al. [16] proposed a strain smoothing technique to stabilize the solutions of the nodal integrated Galerkin mesh-free methods. Liu et al. [17] used the singular edge-based smoothed finite element method (sES-FEM) to analyze the stationary dynamic crack problem.

The extended finite element method (XFEM) developed in 1999 by Belytschko and his coworkers [18,19] has been applied to the dynamic fracture mechanics problems [20–28]. The applications of the XFEM on dynamic cases were mostly focused on dynamic crack growth modeling. By combining the isogeometric analysis (IGA) and the XFEM, the extended isogeometric analysis (XIGA) has been developed. Ghorashi et al. [29] applied the XIGA in the modeling of crack propagation in 2D elastic solids. Yu et al. [30] extended the XIGA to solve the DSIFs of cracked isotropic/orthotropic media under impact loading. Bui et al. [31,32] applied the XIGA to solve dynamic fracture problems in piezoelectric materials and magnetoelectroelastic (MEE) materials.

Singular elements are found to be effective in the modeling of cracks. Existing singular elements mainly fall into two categories, i.e. the Barsoum type element [33,34] and the Benzley type elements [35]. QPE is a typical Barsoum element, which requires the element size to be set properly during the modeling. It is recommended that for a single material crack, the element size R/L should be less than 1/8, where R and L are the element size and a characteristic length. However, dense meshes are necessary around the crack tip. XFEM is a typical example of Benzley element, which is constructed by including additional terms to enrich the deformation mode. As a matter of fact, XFEM requires additional nodal degrees of freedom (DOF) at the affected nodes, which could result in high computational costs. Besides, dense meshes around crack tips are still necessary to get satisfactory solution accuracy.

Symplectic dual approach for elasticity, which was first introduced by Zhong [36], has been applied in many aspects of elasticity [37]. Based on this approach, a series of symplectic analytical singular element (SASE) have been developed to deal with various crack problems in the framework of FEM [38–43]. It was demonstrated that the element size of the SASEs can be chosen more freely, and there is no need of dense meshes near the crack tip. However, the existing SASEs are focusing on static or quasi-static crack problem. In fact, the concept of SASE can also be further extended for dynamic fracture problem.

On the other hand, special treatment is required in solving dynamic problem when applying the SASE to find the DSIFs. On this regard, the precise time domain expanding algorithm (PTDEA) proposed by Yang [44] has been successfully used to deal with dynamic problems [45,46], viscoelastic problems [47,48], and heat transfer problems [49,50]. It was shown that the PTDEA provides better solving efficiency by dividing time domain into a series of sub domains compare with exiting numerical methods. Furthermore, the solving accuracy can be controlled be setting proper time interval in the simulation which can be done automatically. Considering the advantages of the PTDEA in solving dynamic problems, it is employed to deal with the dynamic crack problem in this study. In this way, a new numerical method for dynamic problem by combining the SASE with the PTDEA is proposed, providing better solving efficiency and accuracy. The numerical solution of the DSIFs can be directly obtained without any post-processing. In the numerical examples, it was demonstrated that the method has excellent solving accuracy and efficiency.

2. Fundamental equations

Considering a 2D plane dynamic problem under the Cartesian coordinate system (x, y) as shown in Fig. 1, the constitutive equation about the stresses $\boldsymbol{\sigma} = \{ \sigma_x \quad \sigma_y \quad \tau_{xy} \}^{\mathrm{T}}$ and strains $\boldsymbol{\varepsilon} = \{ \varepsilon_x \quad \varepsilon_y \quad \gamma_{xy} \}^{\mathrm{T}}$ is specified by

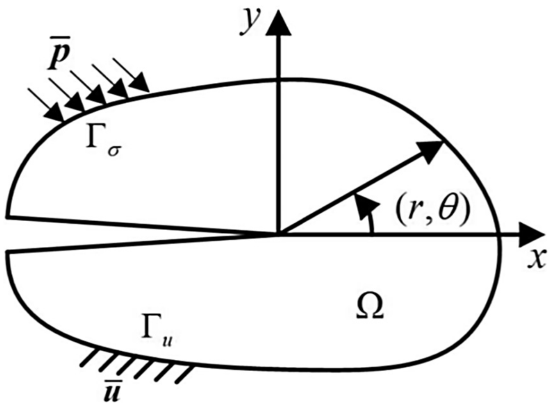

Fig. 1. 2D elastic dynamic problem.

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