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# Crack driving forces for short cracks: The effect of work hardening

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ABSTRACT

The effect of hardening on the crack tip deformation of short cracks is investigated by FEM analysis. The analyses of the results show that the commonly used Shih-Hutchinson and Dowling equations for the calculation of the *J* integral are very good approximations for small and large scale yielding conditions. This method is developed for hardening materials and provides a rough approximation for ideal-plastic or even for softening materials, as long as full scale yielding does not take place. However, in the case of full scale yielding these equations do not provide a satisfying estimation of the *J* integral or crack tip deformation for non-hardening or softening materials, due to a strong dependency on the plastic correction factor. The impact of this phenomenon for nanocrystalline metals or metallic glasses will be pointed out shortly.



## 1. Introduction

As long as the size of the plastic zone or the zone where non-linear deformation in front of the crack takes place, is small compared to the size of the zone where the stress intensity factor *K* dominates the linear elastic stress field, *K* is a useful parameter to describe the behaviour of the crack. In other words *K* can be used as crack driving force.

The deformation in the vicinity of the crack tip is determined by *K*, the Young's modulus *E* and the plastic properties of the material. This loading case is usually donated as small scale yielding regime.

If this size condition is not fulfilled the *J* integral or the $\Delta J$ in the case of cyclic loading are usually used as crack driving forces, because they characterize the stress and strain field near the crack tip. To estimate *J* or $\Delta J$ for crack lengths small compared to the other dimensions of the sample or the component, the approximation of Shih and Hutchinson [1]

$$J = J_{el} + 2 \cdot \pi \cdot f(n) \cdot w_{pl} \cdot a \tag{1}$$

or for the cyclic loading, the extension from Dowling [2] for $n = 1/0.165$ and $f(n) = 1.56$

$$\Delta J = 3.2 \cdot \Delta w_{el} \cdot a + 5.0 \cdot \Delta w_{pl} \cdot a \tag{2}$$

are used, where

$$w_{el} = \frac{\sigma_{el} \cdot \varepsilon_{el}}{2} \tag{3}$$

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or

$$\Delta w_{el} = \frac{\Delta\sigma_{el} \cdot \Delta\varepsilon_{el}}{2} \tag{4}$$

denotes the strain energy density until yielding and

$$\Delta w_{pl} = \Delta\sigma_{pl} \cdot \Delta\varepsilon_{pl} \cdot \frac{n}{1+n} \tag{5}$$

the strain energy density beyond the yield strength.

$J_{el}$ denotes the elastic part of the $J$ integral and can be calculated from $K$ in the regime of small scale yielding and $f(n)$ is a plastic correction depending of the strain hardening exponent $n$. $a$ is the crack length, $\sigma_{el}$ represents the applied stress until and $\sigma_{pl}$ beyond the yield strength.

Shih and Hutchinson [1] assumed a center crack in plane stress conditions and Dowling [2] a half-circular crack in plane stress conditions. For plane strain conditions and a through thickness crack, the prefactors in the equations have to be adapted. Eqs. (1) and (2) permit a very simple estimation of the crack driving force because they only depend on the global elastic and plastic strain energy density and increase linear with the crack length.

It has been shown that this estimation of the crack driving force is exceptionally successful for describing the fatigue crack propagation behaviour for short cracks under low cycle fatigue condition of typical microcrystalline alloys. All these materials exhibit a pronounced cyclic work hardening.

In this paper the effect of hardening on the accuracy of Eqs. (1) and (2) is investigated. Special attention is devoted to the non-hardening or materials with small strain softening. Finite element (FE) analyses are performed on simple tension samples with a trough-thickness crack for the ratios crack length to specimen width ($a/W$) of 2/100, 1/10 and 1/5.

The FE results, the $J$ integral and the crack tip opening displacement (CTOD) are compared with the estimation of Eqs. (1) and (2). Finally the consequences for materials with low strain hardening or softening, like nanocrystalline metals or metallic glasses are shortly discussed.

## 2. Material parameters

In order to provide a non-linear hardening behaviour and the direct connection of the strain hardening exponent $n$ in the analytical calculations to the strain energy density $w$, a Ramberg-Osgood material relation is assumed.

$$\varepsilon = \frac{\sigma}{E} + \alpha \cdot \frac{\sigma}{E} \cdot \left(\frac{\sigma}{\sigma_0}\right)^n \tag{6}$$

All materials have been calculated with a Young's modulus of $E$ = 210 GPa, a Poisson ratio $\nu$ = 0.3 and the material constants $\sigma_0$ = 500 MPa and Eq. (7) for $\alpha$.

$$\alpha = \frac{\varepsilon_{0.2} \cdot E}{\sigma_0} \tag{7}$$

with $\varepsilon_{0.2}$ = 0.002.

The ideal-plastic and small strain softening material behaviours are defined by the following true stress-strain data points, listed in Table 1. For the ideal-plastic material a small gradient is used in order to provide numerical stability.

The ideal-plastic material and the small strain softening material are compared with four different strain hardening materials. The $\sigma$ vs. $\varepsilon$relations of the different materials are shown in Fig. 1, where the colours denote the different material hardening behaviours for $n$ = 5, $n$ = 7, $n$ = 12, $n$ = 15, ideal-plastic and softening behaviour.

## 3. Finite element (FE) simulations

Short crack experiments are often performed on tension or tension-compression specimens. Therefore, the underlying geometry of the following calculations by the FE solver ABAQUS is a displacement-controlled tension sample in plane strain

**Table 1**
Stress-strain data points defining the ideal-plastic and small strain softening material behaviour.

| Material | $\sigma_{pl}$ [MPa] | $\varepsilon_{pl}$ [−] |
|---|---|---|
| Ideal-plastic | 500 | 0 |
| | 520 | 2 |
| Softening | 500 | 0 |
| | 495 | 0.0004 |
| | 495 | 2 |

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