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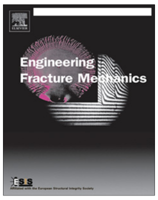

Nonlinear crack opening integral: Mode mixity for finite cracks

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ABSTRACT

In this work the incremental energy release rate is analyzed for single-lap adhesive joints by means of the crack closure/opening integral. To this aim, geometrically nonlinear numerical analyses are performed. For nonlinear elastic analyses the relationship between nodal forces and displacements during nodal load reduction generally cannot be approximated linearly. In the case of a nonlinear relation the forces must be integrated over the displacements in order to calculate the incremental energy release rates. Additionally for a nonlinear analysis the nodal forces and displacements must be transformed into a local nodal coordinate system to separate individual modes and calculate mode mixity. As an alternative, the mode mixity for a finite crack is estimated on basis of the stress state just before or after crack initiation. The mixity values and the resulting critical energy release rates needed for an energy criterion are finally compared to the direct evaluation of the mode separated energy release rates.

1. Introduction

The Incremental Energy Release Rate (IERR) is a quantity well known in the framework of Finite Fracture Mechanics (FFM) [1], it is a measure for the released energy during finite crack formation. Particularly, it is employed within the energy considerations of the coupled stress and energy criterion [1–3] which can be used to assess structural situations with stress concentrations or weak stress singularities present. The basic concept of FFM is founded on the assumption of a spontaneous crack formation with finite size. It was first proposed by Hashin [4]. Physically, the IERR quantifies the energy released during finite crack formation, equivalently to the Differential Energy Realease Rate (DERR) which is well known from Linear Elastic Fracture Mechanics (LEFM) and quantifies the released energy for an infinitesimal crack advancement. Both quantities are averaged over the newly formed crack surface. In contrast to the IERR, the DERR is only applicable to structural situations with initial crack because it vanishes at points of stress concentrations or weak stress singularities. For these situations the IERR can be used beneficially in an energy criterion since it is also applicable for formation of cracks of finite size in a former uncracked structure. In the past, FFM was used successfully for the assessment of weak stress singularities at V-notches [2,3,5–18], adhesive joints [8,19–29], failure of laminates [30–32], cracks at interfaces [33–37] or for the assessment of non-singular stress raisers as U-notches [16,38–40], debonding of inclusions [41–47] and plates with open holes [48–58]. Generally FFM is able to explain size effects such as bond line thickness effects of adhesive joints [59], hole size effects [60,61], notch depth [6] or size effects in laminates [62,63]. These predictions are possible due to the simultaneous fulfillment of a stress and an energy criterion. A comprehensive review on FFM is given by Weissgraeber et al. [1].

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Necessary for all FFM evaluations with a coupled stress and energy criterion, are the stress field and the incremental energy release rate. Here, the estimation of the IERR is the main effort, especially if the individual modes are of interest the crack closure integral (CCI) can be used as done for instance by Hebel and Becker [8]. The crack closure technique is well known from LEFM [64,65], it was introduced by Irwin [66] to calculate the DERR. When using finite element analyses the CCI evaluation is beneficially carried out by means of nodal forces and displacements as proposed by Rybicki [67]. It is known for nonlinear elastic analyses that stress and displacement vectors must be transformed into a suitable crack tip coordinate system for appropriate mode seperation of DERR with CCI [68,69]. With individual modes at hand an energy criterion considering mode mixity can be formulated.

For all purely elastic analyses crack opening and crack closure lead to the same conclusions. However, if dissipation is present, crack opening must be chosen since nonconservative systems lead to path dependent results. In this case, crack opening is the realistic model for a fracture event. Therefore, in the present work the crack opening integral (COI) will be used due to its higher level of generality and reference to reality in estimating the energy released during crack formation.

In this work the COI for a finite crack will be further modified to allow for use in a nonlinear elastic setting. For nonlinear elastic analyses, it is not clear if the stress and displacement vectors during crack opening are related linearly. Since this is a vital issue for the evaluation of COI, it is relevant to take a closer look at these relations. For a linear relation the released energy can be computed using customary multiplication whereas for a nonlinear relation integration of the traction vector over the displacement vector is necessary. In LEFM for infinitesimal crack advancement $\Delta a \rightarrow 0$ the relations are assumed linear and multiplication is used. In this work it will be investigated if the COI can be evaluated by multiplication or must be integrated due to nonlinear effects during finite crack formation. Moreover if the crack opening displacements assume large values integration may be useful in a local coordinate system. Therewith it is assured that the individual contributions are strictly split into normal and tangential direction with respect to the crack surface. Nonlinear elastic behavior can result from geometric nonlinearities as e.g. in single-lap adhesive joints or from nonlinear elastic material behavior as e.g. silicone adhesives. This is the reason why the nonlinear crack opening integral (NCOI) will be compared to a linearized crack opening integral for single-lap joint geometries with linear elastic or hyperelastic material behavior. Finally an energy criterion considering mode mixity will be formulated to be used for FFM.

In the remaining part of this introduction additional methods for estimation of the energy release rate and mode mixity will be given. This is done to give a quick overview of the topic and to provide reference values or alternative estimation methods for the individual modes.

Many authors determine the IERR by simply dividing the change in potential energy (uncracked and cracked) by the newly formed crack surface. Hebel [70], Carrere et al. [71] and Weissgraeber et al. [29] calcluate the potential energy difference numerically. Garcia and Leguillon [10] use a semi-analytical approach requiring an additional numerical simulation to provide for the potential energy difference. The employed method, asymptotic matching, is described by Leguillon [72] and also Yosibash et al. [18]. One drawback of the potential energy approach is that separation of crack opening modes is not straightforward.

Another approach is to integrate the DERR. That enables to separate the crack opening modes if the DERR is available for distinct modes. Carrere et al. [71] calculate the DERR with the Modified Virtual Crack Closure Technique (MVCCT) numerically. Cornetti et al. [73], Muñoz-Reja et al. [74], Weissgraeber and Becker [28] and Stein et al. [26] analytically evaluate the DERR for a weak interface model with the formulas provided by Carpinteri et al. [75], Lenci [76] or Krenk [77]. One drawback of the integration method is that it is only applicable to cracks with one crack tip. For cracks with two tips, originating inside a body and not from a boundary, integration of the DERR is questionable.

For linear elastic material behavior, it is also possible to compute the IERR by means of the stress intensity factors or to use the latter directly in the energy criterion as in Weissgraeber et al. [48], Cornetti et al. [3], Hebel et al. [30] or Leguillon et al. [19].

The most direct approach to calculate the IERR and mode mixity for a finite crack is the direct evaluation of the crack closure/crack opening integral (CCI/COI). Hell et al. [21] investigate the relation between DERR and IERR for the linear elastic case. Hebel [8,70] presents formulas to calculate the IERR mode-separated with the CCI but uses the total IERR for the energy criterion.

In contrast to infinitesimal crack growth, for finite crack growth, the type of loading has an influence on the released energy. For fixed-load boundary conditions, the released energy is somewhat higher than for fixed displacement boundary conditions [49]. For the case of infinitesimal crack growth, the energy release rate becomes independent of the loading conditions [64,65]. Analyzing experiments in which finite crack growth is expected it is important to use the correct experimental loading conditions [78] to provide accurate predictions.

The mode mixity for finite crack formation can be calculated directly on basis of energy considerations or by considering the stress state just before or directly after crack initiation. For instance the mode separated incremental energy release rates [71] can give an estimate of the mode mixity. Otherwise the DERR of the newly formed finite crack can be used. The second energy approach is again not feasible for a crack inside a body with two crack tips.

Furthermore, the mixity is sometimes defined as the angle between the traction vector and the normal to the plane it acts upon. This can be evaluated at a small distance ahead of the crack tip introducing an additional length scale, see Mantič [79] or Hutchinson and Suo [80], or directly at the crack tip or notch root, Hebel [70]. Mantič computes the mixity based on the stresses after crack formation whereas Hebel uses the stresses at the notch root prior to crack initiation.