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# A discussion on failure criteria for ordinary state-based peridynamics

Daniele Dipasquale *, Giulia Sarego, Mirco Zaccariotto, Ugo Galvanetto

*Department of Industrial Engineering, University of Padova, Via Venezia 1, Padova 35131, Italy*
*Centre of Studies and Activities for Space (CISAS) - "G.Colombo", Via Venezia 15, Padova 35131, Italy*

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ABSTRACT

Peridynamics is a recently proposed nonlocal theory of continuum particularly suitable to describe fracture mechanics, since it employs an integral formulation which remains valid whenever discontinuities are present. In this paper, mixed-mode fracture cases are taken into consideration to numerically evaluate three failure criteria of the nonlocal interaction, called bond, between two material particles. These criteria are related to the maximum stretch of a bond or to the maximum energy that can be stored in a deformed bond. Their advantages and shortcomings are presented and discussed.



## 1. Introduction

Nonlocal linear elasticity and microcontinua have been studied for many decades [1–4] in an effort to account for long-range effects. PeriDynamics (PD) [5] is included in the set of nonlocal theories, since two body particles can interact with each other even when separated by a finite distance, provided it is smaller than a limit distance called *horizon*.

Differently from the classical theory, peridynamics uses an integral formulation to compute the forces on a material particle and such formulation remains valid even if discontinuities are present. A unique framework of mathematical equations can be used both when discontinuities are involved and when they are not, thanks to the integral formulation. In this way, not only crack propagation direction does not have to be known *a priori*, but also crack initiation points do not have to be located in advance, since both phenomena are inherently captured.

The fundamental element of this theory is based on the long-range interaction between pairs of material points of the body, called *bond*. The Bond-Based peridynamics (BBPD) formulation was the first proposed by Silling [6]. The peridynamic force exerted through the bond can be uniquely described by the relative initial and current position vectors between the two points identifying the bond, provided that the material properties have been associated to these points: no further information from their surrounding points or bonds is required. In order to overcome the shortcoming intrinsically introduced by BBPD formulation, e.g. the fixed Poisson's ratio (see [7]), a reformulation of peridynamics has been introduced in 2007 [8] and more extensively described in 2010 [9] called State-Based peridynamics (SBPD). Bonds can influence each other thanks to mathematical functions called states. In this way, limitations are overcome, and unlike BBPD, more general behavior and complex phenomena can be described. Besides, the definitions of (nonlocal) strain and stress may be included, breaking the barrier to use the classical constitutive material models described in terms of a stress tensor in PD formulation.

* Corresponding author at: Department of Industrial Engineering, University of Padova, Via Venezia 1, Padova 35131, Italy.
*E-mail addresses:* daniele.dipasquale@studenti.unipd.it (D. Dipasquale), ugo.galvanetto@unipd.it (U. Galvanetto).

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An important attribute introduced by peridynamics is the unambiguous definition of damage at a material point of the body as a consequence of the progressive permanent rupture of the network of bonds. Several failure criteria can be found in literature in order to decide when a bond has to be broken, the simplest one is based on bond elongation [10], namely a bond breaks when a critical elongation is reached. This criterion was introduced in BBPD for dealing mainly with mode I fracture in both homogenous [11–13] and heterogeneous [14] brittle materials. Although bonds can break under a shear deformation, this criterion only involves the volumetric part of the deformation being not sensitive to the deviatoric part. Hence, it is not suitable to simulate the fracture commonly observed in elasto/visco-plastic materials. In order to overcome this shortcoming, a criterion based on the energy stored in a bond was proposed in **[15]**, while a similar energy based failure criterion was employed in [16] by including the J-integral computation around the crack tip of the crack. Both criteria have been validated by investigating only mode I fracture [16,17]. More complex failure criteria are applied in the frame of the "damage correspondence model", namely the damage models adopted in the classical theory of mechanics are directly incorporated in the peridynamic formulation, such as the equivalent strain criterion presented in [18] and the standard Johnson-Cook damage model presented in [19]. So far, to the authors' knowledge no studies have been carried out by using SBPD for comparing the existing failure criteria in problems concerning mixed-mode dynamic brittle fracture. Therefore, in this work new aspects are treated: several failure criteria are applied in the state-based 2D framework with the aim to investigate which one is more suitable to capture the failure patterns experimentally observed in mixed-mode of brittle fracture. In addition, we modify the failure criterion presented in [16], reducing the computational cost of the simulations.

In this paper we briefly describe in Section 2 the fundamentals of ordinary state-based peridynamic theory and its numerical discretisation, Section 3 contains a discussion of the proposed failure criteria to be applied in ordinary state-based peridynamics. Section 4 presents three numerical examples that have been analyzed to evaluate the failure criteria and finally the relevant conclusions are drawn.

## 2. The peridynamic theory

### 2.1. Ordinary state-based peridynamic

The peridynamic theory is a formulation of the equations of solid mechanics introduced by Silling and coworkers in [8], where nonlocal interactions between particles within a limit distance called *horizon* are defined. These nonlocal interactions may influence each other through functions called peridynamic states. A state is any function of all the bonds within the neighborhood. In the equation of motion the divergence of the stress tensor, typical of classical mechanics, is substituted in the SBPD formulation by the integral of the internal forces

$$\rho(\mathbf{x})\ddot{\mathbf{y}}(\mathbf{x},t) = \int_{\mathcal{H}} \{\underline{T}[\mathbf{x},t]\langle \mathbf{p}-\mathbf{x}\rangle - \underline{T}[\mathbf{p},t]\langle \mathbf{x}-\mathbf{p}\rangle\}\, dV_p + \mathbf{b}(\mathbf{x},t) \tag{1}$$

where $\rho(\mathbf{x})$ is the mass density at the point $\mathbf{x}, \ddot{\mathbf{y}}(\mathbf{x},t)$ its current acceleration vector (i.e. $\mathbf{y}(\mathbf{x},t)$ is its current position vector, see Fig. 1), $\underline{T}[\mathbf{x},t]\langle \mathbf{p}-\mathbf{x}\rangle$ is the force state at time $t$, applied to the bond $\langle \mathbf{p}-\mathbf{x}\rangle$ and at point $\mathbf{x}, dV_p$ is the infinitesimal volume associated to $\mathbf{p}, \mathcal{H}$ the neighborhood of the point $\mathbf{x}$ containing all the points $\mathbf{p}$ interacting with $\mathbf{x}$.

A peridynamic state is a mathematical object introduced for mapping vectors called bonds defined as $\xi = \mathbf{p}-\mathbf{x}$ into general quantities such as a scalar, a vector or a tensor. Let $\underline{X}$ define the *reference state* for mapping any bond to its initial position vector as

$$\underline{X}\langle \mathbf{p}-\mathbf{x}\rangle = \mathbf{p}-\mathbf{x} \tag{2}$$

defining the *displacement state* $\underline{U}$ to associate the bond to its displacement (i.e. the relative displacement between the two points) as

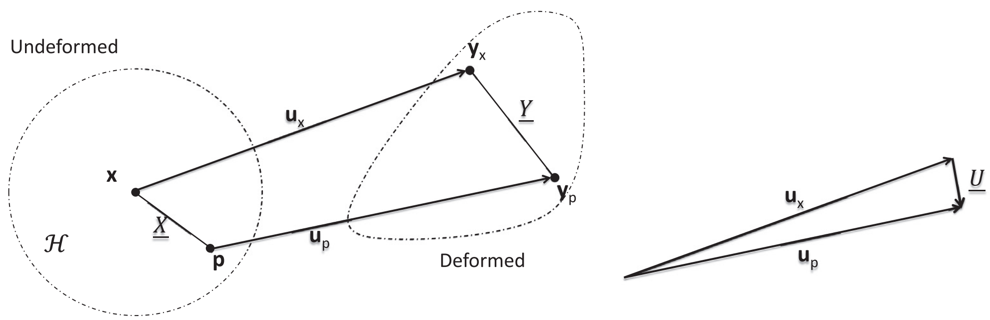


**Fig. 1.** Deformation of the bond involved in Eq. (1) and relation with the reference state $\underline{X}$, the deformation state $\underline{Y}$ and the displacement state $\underline{U}$.