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A two-scale generalized finite element method for interaction and coalescence of multiple crack surfaces

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ABSTRACT

This paper presents the application of a two-scale generalized finite element method (*GFEM*) which allows for static fracture analyses as well as fatigue crack propagation simulations involving the interaction of multiple crack surfaces on fixed, coarse finite element (FE) meshes. The approach is based on the use of numerically-generated enrichment functions computed on-the-fly through the use of locally-defined boundary value problems (BVPs) in the regions of existing mechanically-short cracks. The two-scale *GFEM* approach is verified against analytical reference solutions as well as alternative numerical approaches for crack interaction problems, including the coalescence of multiple crack surfaces. The numerical examples demonstrate the ability of the proposed approach to deliver accurate results even in scenarios involving multiple, interacting discontinuities contained within a single computational element. The proposed approach is also applied to a crack shielding/crack arrest problem involving two propagating crack surfaces in a representative panel model similar in complexity to that which may be of interest to the aerospace community.

1. Introduction

The pursuit of accurate fatigue life predictions for structural components has been an active area of research in the civil, mechanical and aerospace communities for decades. From a numerical modeling standpoint, fatigue propagation simulations have been broadly performed for many years using standard FE approaches [1–5]. In a standard FE approach, the crack surface itself is modeled via meshes with double nodes to define the discontinuity and degenerate quarter-point elements in the crack tip region to model the strong gradients and singularity developing in the stress field in this region. Conventional FE approaches to fracture problems often lead to large problem sizes requiring significant amounts of computational resources [6,7]. There has been much work in the development of adaptive meshing techniques [8,9,7,10–14] which are aimed at the generation of highly graded meshes in crack tip regions, often employing crack front template meshes to help resolve the singularities. The use of highly graded meshes in general may lead to conditioning issues in the resulting system of equations. Additionally, crack front template meshes which utilize brick elements can become very difficult to construct in the case of evolving three-dimensional crack surfaces [11] and user intervention may be required to generate a suitable mesh [15]. With adaptive meshing approaches, care must be taken to ensure a well-structured mesh in the crack tip regions

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Nomenclature

a	crack length
BVP	boundary value problem
C	parameter in the Paris-Erdogan equation
$\frac{da}{dN}$	crack growth rate in length/cycle
E	Young's Modulus
$GFEM$	generalized finite element method
$GFEM^{gl}$	generalized finite element method with global-local enrichment functions
$H^1(\omega_\alpha)$	first order Hilbert space defined on ω_α
K_I, K_{II}, K_{III}	mode I, II, and III stress intensity factors
K_{eq}	equivalent stress intensity factor
$L_{\alpha i}$	enrichment function defined on ω_α
m	exponent in the Paris-Erdogan equation
N	number of load cycles
R	load ratio
r	initial crack radius
$SIFs$	stress intensity factors
$\boldsymbol{u}$	displacement field
$\boldsymbol{u}_{hp}$	finite element approximation of $\boldsymbol{u}$
$\boldsymbol{x}$	point in space
$XFEM$	extended finite element method
α	index of node in the finite element mesh
Δa_{max}	maximum crack front increment
ΔK	stress intensity factor range
$\Delta K_{eq_{max}}$	maximum equivalent stress intensity factor range
ν	Poisson's ratio
θ	crack front kinking angle
σ'_1	maximum principal stress
$\sigma_\theta, \sigma_z, \tau_{\theta_z}$	components of the stress tensor
ψ	crack front twisting angle
φ_α	finite element partition of unity function
$\phi_{\alpha i}$	generalized finite element shape function
χ_α	local approximation space on ω_α
Ω	solid, finite element domain
ω_α	set of elements connected to node α

throughout the course of a simulation so as to avoid, for instance, elements with poor aspect ratios which may adversely impact the resulting solution quality [13].

There are many existing numerical techniques which are aimed at avoiding the aforementioned meshing requirements and computational costs associated with conventional FE approaches to fracture analyses, while still maintaining sufficient levels of accuracy. The generalized/extended finite element method (*G/XFEM*) [16–21] is one such technique, which has been actively developed over the past ten years or so. As opposed to standard FE approaches which require the generation of a volume mesh which 'fits' the crack surface at every crack step [7,3,12,10,11], the *G/XFEM* is based upon the notion of using enrichment functions to accurately model the crack surface as it propagates throughout the course of a simulation. In this manner the *G/XFEM* is able to alleviate some of the potentially cumbersome modeling considerations, and associated computational costs often encountered with a standard FE approach.

To further complicate the matter, in a high cycle fatigue framework, much of the fatigue life is spent in the crack initiation phase. In this phase multiple, diffuse discontinuity surfaces propagate and link up with one another in the formation of a single, dominant flaw, which eventually cause the ultimate fatigue failure of a component. A key element of a numerical tool appropriate for use in such a framework is the ability to simulate the interactions and eventual coalescence of multiple crack surfaces. The problem of crack coalescence has previously been investigated in the standard *G/XFEM* context in [22,23]. In the latter it was noted that there are issues with the enrichment strategy used in the *XFEM* when crack fronts approach one another due to the use of an implicit crack surface representation. Both approaches relied upon available analytical enrichment functions, defined separately for each crack under consideration, up until the point of coalescence.

A potential limitation of the standard *G/XFEM* approach is the *a priori* requirement of enrichment functions which can accurately model the physics of the problem at hand. Unfortunately, accurate closed-form enrichment functions