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A benchmark fracture mechanics solution for a two-dimensional eigenstrain problem considering residual stress, the stress intensity factor, and superposition

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ABSTRACT

Eigenstrain is a distributed strain field considered in mechanics that is particularly helpful in evaluating residual stress fields in the finite element method, and estimating the stress intensity factor due to residual stress in cracked components. The objective of this paper is to provide a solution for a simple eigenstrain problem in a two-dimensional rectangular domain that can serve as a benchmark for validation of fracture mechanics analysis methods. The solution provides residual stress fields and the stress intensity factor for a single edge crack as a function of crack size. Documenting the benchmark provides opportunities to demonstrate the correlation of different means to determine the stress intensity factor and to highlight details in implementing stress intensity factor calculations.

1. Introduction

Residual stresses are known to have an important influence on fatigue life [1]. Significant research efforts have been devoted to prediction of residual stresses from processes that enhance fatigue performance of materials by introducing compressive residual stresses [2–4]. One of the leading methods to analyze residual stresses is the eigenstrain method [5], which reconstructs the complete residual stress field based on an elastic finite element analysis that includes the initial permanent strain (i.e., the eigenstrain field) causing the residual stress. The eigenstrain field is determined using process modeling or experimental measurements. Because fatigue failure comprises the initiation and propagation of cracks, the stress intensity factor of fracture mechanics (and the related J-integral) is important in fatigue life prediction.

Calculation of the stress intensity factor is a conventional technology, but care is required when assessing the combined effects of applied loads and residual stresses. Stress intensity factors for applied loads are typically computed using available handbook solutions for simple cases, or using finite element or boundary element methods for complex cases. For residual stresses, stress intensity factors can be determined using the weight function method for simple cases, or the more general finite element or boundary element methods for complex cases. The principle of superposition is typically used to combine the effects of residual stresses and cyclic applied loads when assessing fatigue and fracture behavior.

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Nomenclature

a	crack length
A	domain enclosed by the closed contour $C + C_+ + \Gamma + C_-$
$\mathbf{B}$	pre-logarithmic energy factor matrix (diagonal for homogeneous, isotropic materials)
C	external contour enclosing Γ used in computing J
C_-	portion of A that runs along the lower crack face
C_+	portion of A that runs along the upper crack face
E	Young's modulus
$\bar{E}$	material property, E for plane stress, $E/(1-\nu^2)$ for plane strain
F_j	functions of crack size related to the weight function
G	shear modulus
H	plate height
$\mathbf{H}$, H_{ij}	second order tensor used in J-integral calculation
J	J-integral
K	stress intensity factor
K_I	mode I stress intensity factor
K_{II}	mode II stress intensity factor
K_{III}	mode III stress intensity factor
$m(a,x)$	weight function for stress intensity factor calculation
$\mathbf{m}$	outward normal to the domain enclosed by the closed contour A
$\mathbf{n}$	outward normal to Γ
$\mathbf{q}$	unit vector in the virtual crack extension direction
$\bar{\mathbf{q}}$	sufficiently smooth weighting function within the closed contour A
S	normalization parameter (0.001E)
$\mathbf{t}$	surface traction on the crack surfaces
$\mathbf{u}$	displacement vector used in computing J
u	crack face displacement along y-direction
u_r	reference crack face displacement used in weight function derivation
U	elastic strain energy density
W	width of the plate
x	spatial coordinate along the crack line
β_i	functions of normalized crack size used in the weight function
δ_{ij}	Kronecker-delta
ε^*	eigenstrain field
$\varepsilon^*_{xx}, \varepsilon^*_{yy}, \varepsilon^*_{xy}$	eigenstrain components
ν	Poisson's ratio
$\boldsymbol{\sigma}$	stress tensor
$\sigma(x)$	stress field acting to open the crack
σ_{TOT}	total residual stress
σ^*	residual stress due to eigenstrain
σ_{EQ}	residual stress arising to satisfy equilibrium
Γ	contour enclosing the crack tip

1.1. The eigenstrain method

The term eigenstrain was initially suggested by Mura [6], and eigenstrain methods have been discussed by several authors in the context of assessing residual stress fields. According to DeWald and Hill [7], eigenstrain can be considered an inelastic strain distribution that causes a given residual stress field. It is an incompatible strain field that does not satisfy geometric (strain) compatibility, and leads to a total strain field that satisfies mechanical equilibrium through an induced residual stress field. A previous study demonstrates the use of eigenstrain methods to simplify the estimation of stress fields from compressive residual stress surface treatments [7,8]. In that work, a simplified eigenstrain field was determined from limited residual stress measurement data, and finite element models where used to determine the full residual stress field from the eigenstrain. Coratella et al. [9] provide a recent validation of that earlier work in aerospace aluminum alloy (7050-T7451) samples with residual stresses from laser shock peening (LSP). Luckhood, Jun, and Korsunsky [10] have used the eigenstrain method to analyze residual stresses measured in friction stir welds. Kartal et al. [11] illustrate the use of eigenstrain for determination of microscale residual stresses near an inclusion in a nickel alloy. A closely related approach is to use non-linear process models, rather than measurements, to approximate the eigenstrain field produced by a given process. For example, Achintha and Nowell [12], and Hu and Grandhi [13], have used non-linear (finite element) process