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Macro-scale fracture analysis of isothermal composites: Theory and seven applications

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ABSTRACT

Starting with the macroscopic energy balance and the macroscopic entropy inequality, a lower bound is derived for the critical strain energy at which an isothermal composite or multiphase material fractures or fails. It relates the work done on the body, or the strain energy created in the body, to the new fracture surface area created. No assumption is made about the number or configurations of these fractures. Using previously published experimental observations by others, seven examples are developed to demonstrate how this inequality can be used to calculate bounds that can be compared with experimental observations. Six involve carbon fiber/epoxy aerospace laminates, and one an oil/gas producing sandstone. No adjustable parameters or history matching are employed.

1. Introduction

There are several recent reviews of fracture [1–6]. For this reason, we give only a limited discussion of the literature. We will focus our attention here on the use of the second law (entropy inequality) in discussions of fracture.

Griffith [7] is widely viewed as the father of modern fracture mechanics. His starting point was the theorem of minimum energy [8, Exercise 4.10.3-2], which assumes that a body is at equilibrium. Broutman and Koboyashi [9] (see also Anderson [6]) note that "Griffith assumed the presence of very small cracks in the material and made use of Inglis' calculation of stresses by regarding the cracks as very flat elliptical holes. . . .In the case of a thin elastic plate (plane stress) with a very flat elliptic crack . . .under uniaxial tension, Griffith's assumptions lead to the famous Griffith equation . . .." As we do in what follows, Griffith assumed in addition that the body was isothermal, that the surface tension was a constant, and that a cusp was formed at the fracture edge.

Working in the spirit of Gurtin [10], Slattery et al. [11] and Fu and Slattery [12] (see also [13,14]) did recognize interfacial effects to find for a single-phase, single-component body undergoing mode I fracture that the rate at which work is done by the body on the surroundings at the fracture edge, or the critical energy release rate, is

$$G_c \equiv 4\gamma \tag{1}$$

Here γ is the thermodynamic surface tension in the fracture surface or, in effect, the force per unit length of line that each fracture surface exerts on the fracture edge. We show in Appendix A that this result is not limited to mode I fractures.

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Nomenclature

$\widehat{A}$	Helmholtz free energy per unit mass
$\widehat{A}^{(\sigma)}$	surface Helmholtz free energy per unit mass
$\mathbf{b}$	body force per unit volume
$\mathbf{b}^{(fe)}$	body force per unit length of fracture edge
$C^{(fe)}$	fracture edge
E_{input}	energy input
G_{Ic}	critical energy release rate for a mode I fracture
K_{Ic}	fracture toughness or stress intensity factor for a mode I fracture
$\mathbf{n}$	the unit normal to S that is outwardly directed
$\widehat{S}$	entropy per unit mass
$\widehat{S}^{(\sigma)}$	surface entropy per unit mass
t	time
$\widehat{U}$	internal energy per unit mass
$\widehat{U}^{(\sigma)}$	surface internal energy per unit mass
$\mathbf{v}$	velocity
$\mathbf{v}^{(\sigma)}$	surface velocity
$\mathbf{v}^{(fe)}$	velocity of the fracture edge
dA	area integration
ds	line integration
dV	volume integration
$d_{(\mathbf{v})}/dt$	the derivative with respect to time following a material particle
$d_{(\mathbf{v}^{(\sigma)})}/dt$	the derivative with respect to time following a surface particle
γ	thermodynamic surface tension
$\boldsymbol{\nu}$	the unit vector that is normal to the fracture edge, tangent to all three interfaces, and outwardly directed with respect to the body
ρ	mass density
$\rho^{(\sigma)}$	surface mass density

1.1. Macroscopic energy balance and macroscopic entropy inequality

The macroscopic balances have been discussed by many, perhaps most notably by [15]. Slattery et al. [8] included interfacial effects as well as the macroscopic entropy inequality. To our knowledge, there has been no previous discussion along these lines for fracture.

2. Objective

Our objective is to develop a lower bound for the internal strain energy at which an isothermal composite or multiphase body fractures (fails). We will make the following assumptions.

1. We will assume that fracture is a stochastic process and that the area of newly created fracture surface does not have an experimentally repeatable value.
2. We will assume that the body has a uniform temperature. There is no mass transfer across the boundary of the body, and the macroscopic mass balance is satisfied identically.
3. Following Slattery et al. [11], we will assume that the fracture forms a cusp at the curve representing the fracture edge.
4. We will neglect the effects of gravity.

We will make no explicit assumptions about material behavior. However, the material behavior is taken into account through the critical energy release rate. Because we are using macroscopic relations, this will be the critical energy release rate for a single phase representation of the multiphase body.

There are no assumptions made about the internal details of the multiphase body. For example we say nothing about the layout of the fibers in a laminate. These internal details, however, do influence the critical energy release rate.

The ultimate justification of these assumptions will be in the agreement between the theory and the experimental data in each of the seven examples of Section 4. Note also that in comparing theory with experiment we will use only parameters from the literature; we will not use adjustable parameters or history matching.