Engineering Fracture Mechanics xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

An analytical solution for evaluating the effect of steel bars in cracked concrete

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ARTICLE INFO

Article history:
Received 30 November 2015
Received in revised form 13 April 2016
Accepted 3 June 2016
Available online xxxx

Keywords:
Analytical solution
Nonlinear analysis
Reinforced concrete
Load-carrying capacity
Constitutive modeling

ABSTRACT

An analytical solution is proposed for investigating the mechanical behavior of a crack in steel-reinforced concrete. In this solution, an elastic–plastic constitutive law was assumed to satisfactorily describe the behavior of the steel bar, and the bridging traction was deduced from the deformation of steels. The curves of the load versus crack mouth opening displacement predicted by the proposed method could be fitted to experimental results. It was analytically demonstrated that the ultimate load-carrying capacity calculated by the developed method is not sensitive to the shape of the softening curve for a large beam with high reinforcement ratio.

1. Introduction

Cementitious-matrix materials are very brittle and have low tensile strength. In order to improve these characteristics, fibers or steel bars with high tensile strength and large strain capacity are buried in the matrix, especially in the tensile stress zone or region in which cracking is expected [1,2]. In fact, experimental tests and analytical studies showed that the addition of fibers or reinforcement to conventional concrete can suppress propagation and widening of cracks and reduce long-term deflection of structures under loading as well as restrained conditions [3,4]. However, the fibers or steel bars also alter the mechanical response of the matrix, especially when the level of stress is high enough to generate cracks. To address this problem, a number of experimental and analytical studies have been conducted to analyze this cracking process and account for the stress redistribution and mechanism of load transfer between the matrix and the reinforcement [5,6].

Many test programs have focused on the effects of basic parameters on the fracture behavior, which include specimen size, cross-sectional shape, steel arrangement, steel–cement ratio, steel yield strength, bond–slip properties, loading rate, etc. [2–8]. Lameiras et al. [2] carried out an experimental investigation to determine the tensile softening constitutive behavior of fiber reinforced concrete and proposed a modified splitting test to assess the post-cracking behavior. Sagar and Rao [7] studied the effect of the loading rate and showed that when the loading rate is higher, quick crack development leads to rapid fluctuations and the concrete becomes more brittle. Tiberti et al. [5] investigated the ability of steels to control cracks by analyzing more than 90 tension tests on reinforced concrete prisms with different sizes, reinforcement ratios, and concrete strengths. In particular, it was found from Alam's experimental results [9] that EN Eurocodes of the European Committee for Standardization underestimate the crack width and crack spacing. The measured values of crack width significantly depend on the structural size, which is not explained by the Eurocode crack width equation. With regard to the failure mode, Carpinteri et al. [6] concluded that the collapse of a reinforced structure resulting from the

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http://dx.doi.org/10.1016/j.engfracmech.2016.06.002
0013-7944/

Please cite this article in press as: Mi Z et al. An analytical solution for evaluating the effect of steel bars in cracked concrete. Engng Fract Mech (2016), http://dx.doi.org/10.1016/j.engfracmech.2016.06.002

Nomenclature

a	effective crack length
a_0	initial crack length
A_{si}	cross-sectional area of the i-th reinforcement
b	width of specimen
$CMOD$	crack mouth opening displacement
E	Young's modulus of concrete
E_{si}	Young's modulus of the i-th reinforcement
f_t	tensile strength of plain concrete
h	height of specimen
K_I	global stress intensity factor
K_{IC}^{ini}	initiation fracture toughness
K_{IF}	stress intensity factor due to the reinforcing bar
K_{IM}	stress intensity factor due to the bending moment or external load
$K_{I\sigma}$	stress intensity factor due to the cohesive stress
$m(x,a)$	weight function
M	external applied bending moment
M_1, M_2, M_3, M_4	parameters of weight function
P	external applied load
S	span of specimen
x_i	distances from the center of the i-th reinforcement to lower edge of beam
β	material constants for power softening function
ε_{syi}	yield strain of the i-th reinforcement
$\sigma_2(x)$	cohesive stress
σ_{syi}	yield strength of the i-th reinforcement
$\omega(x)$	crack opening displacement
$\omega(x_i)$	crack opening displacement at the i-th reinforcement
ω_0	crack opening displacement at the tip of preformed crack
ω_c	critical value of the crack opening displacement at which the value $\sigma = 0$
ω_{i0}	initial crack width at the i-th reinforcement

ductile-to-brittle transition is dependent on the specimen size, which indicates that this failure mode is related to the fracture process zone in concrete. Meanwhile, Chen et al. [10] also discovered that the axial load ratio and shear span ratio are significant factors of the ductility and crack opening width of reinforced concrete structures, while the longitudinal steel ratio has negligible influence on the column ductility, but exhibits stronger effect on the beam ductility. It must be noted that Ruiz et al. [8] observed that reinforced concrete beams exhibit a shape effect on the maximum load so that a horizontal arrangement of steel bars would trigger a secondary load peak after concrete cracking; this finding shows that the shape effect is comparable to the size effect. Regarding the large-deformation behavior of reinforced concrete specimens, Su et al. [11] concluded that a significant improvement of the flexural strength of a reinforced concrete beam results from the compressive arch action, which could be influenced by the flexural reinforcement ratio and span-height ratio of the beam. In addition, an interesting phenomenon was observed by Tschegg et al. [4], who investigated the differences in dissipated energy under biaxial tension-compression and uniaxial tension load of reinforced concretes using the wedge splitting test. Their results showed that the specific fracture energy for a biaxial tension-compression load is on average 20–30% lower compared to that of a uniaxial tension load, which is attributed to the damage mechanism of the concrete matrix and deterioration of the aggregate-cement-paste interfaces in case the section is additionally loaded with compression stresses.

Fracture models for steel reinforced concrete have also been proposed for analysis of the cracking process, stress redistribution, and load-transfer mechanism between the matrix and the reinforcement. These models can be classified into three groups, namely the bridged crack model [6,12], cohesive crack model [13,14], and sectional moment-curvature model [15,16].

The bridged crack model was originally proposed by Carpinteri et al. and used to determine the minimum size-dependent amount of reinforcement [6]. The longitudinal reinforcements were considered as a pair of concentrated force applied to the edges of a crack. The model was later amended by modifying the constitutive flexural response of unreinforced concrete with cohesive closing stresses and extending the model to two different types of reinforcements (namely longitudinal bars and fibers) simultaneously distributed in the cementitious matrix [17]. A remarkable feature of the reformulated model is the application of both the equilibrium and compatibility conditions to the cracked section, and the model can be applied to a wide range of composite materials. With regard to the behavior of concrete in compression, Hillerborg [13] originally proposed a model, based on the concept of strain localization, for analyzing the bending of over-reinforced concrete beams. This model was later improved by Carpinteri et al. [18] through the introduction of the overlapping-crack model, which is analogous to the cohesive crack model and allows for description of the concrete softening due to stress in the compression zone. The improved model could describe both cracking and crushing growth during loading processes in reinforced concrete