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Periodic cracks in an infinite electrostrictive plane under the influence of a uniform remote electric field

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ABSTRACT

We present a rigorous treatment of the problem of collinear periodic cracks in an infinite electrostrictive plane subjected to plane strain deformation in the presence of a uniform remote in-plane electric field. Using complex variable methods, exact electric and stress fields in the electrostrictive plane are obtained for both permeable and impermeable cracks. We show that the ratio of the mode-I stress intensity factors for periodic cracks to those for a single crack is determined by only the ratio of the period of the periodic crack-system to the crack length. Additionally, we find that the stress intensity factors for periodic cracks and the stress field in the proximity of periodic cracks can be treated essentially as those for a single crack when the period exceeds four times the crack length.

1. Introduction

Electrostriction is a property of electrical insulators (e.g. dielectrics) that causes them to deform when subjected to an electric field. Electrostrictive materials are particularly attractive for use in smart structures since they react to electric fields very quickly (as required, for example, in high-speed control circuits). Unfortunately, micro-cracks often develop in electrostrictive materials during the manufacturing process as well as a result of damage accumulation through sustained fatigue. In an effort to predict the reliability of electrostrictive devices, considerable attention has been given to the fracture behavior of electrostrictive materials weakened by cracks. For example, the study of fracture of piezoelectric materials (a subset of electrostrictive materials) has attracted significant attention in the literature (see, for example, [1–8]). In contrast to piezoelectricity, electrostriction is almost exclusively a nonlinear effect, especially in most electroactive polymers. The early development of the theory of electrostriction can be found in [9–11]. In the past decade, the theory has attracted considerable attention with the development of a new formulation and innovative variational principles to describe the electrostrictive mechanics of finite deformations (see, for example, Dorfmann and Ogden [12,13], McMeeking et al. [14,15], Kuang [16,17], Suo et al. [18], Bustamante et al. [19,20] and Skatulla et al. [21]).

The mathematical difficulties associated with the nonlinear framework of electrostriction led Knops [22] and Smith and Warren [23] to adopt a simplified dielectric response in which the contribution of the strain field to the electric displacement was taken as insignificant (see also [24,25]). Under this assumption, the authors used complex variable methods to solve the electric field-induced stress problem. The same method was employed by McMeeking [26] and Beom et al. [27] to investigate the electrostrictive stress in the vicinity of crack tips in a cracked solid. A further limitation of the work of Knops [22]

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and Smith and Warren [23] relates to the absence of any consideration of the Maxwell stress in the electrostrictive body. Recognizing the importance of the Maxwell stress in the fracture analysis of electrostrictive materials [29], Jiang and Kuang [28] presented a revised version of the complex variable method to incorporate the Maxwell stress. Based on the revised formulation from [28], Gao et al. [30–32] established analytical solutions corresponding to problems involving a single crack and a series of collinear cracks within an electrostricitve solid. To the authors' knowledge, however, the solution for the more general and practical problem in which periodic cracks are present in an electrostrictive material remains absent from the literature. In this paper, we address this deficiency and consider the influence of periodic cracks in an electrostrictive material under the influence of a uniform remote electric field.

The paper is organized as follows. Based on the revised formulation in [28], our fundamental equations and boundary conditions for the present problem are given in Sections 2 and 3, respectively. Closed-form solutions to the electric and stress fields are derived for permeable and impermeable cracks in Section 4. Finally, we summarize our findings in Section 5.

2. Problem formulation

As shown in Fig. 1, we refer to a Cartesian coordinate system (x, y) and consider periodic collinear cracks lying on the x-axis in an isotropic electrostrictive solid under plane-strain deformation subjected to a uniform remote electric field E_y^∞ along the positive y-axis. The crack faces are assumed to be either permeable or impermeable to the electric field.

We consider a representative strip with a single crack as shown in Fig. 2. Following a simplified linear dielectric response (see [22–25]), the components (E_x, E_y) of electric field and (D_x, D_y) of electric displacement are described in terms of a single complex function $f(z)$ $(z = x + iy)$ defined in the cracked strip by

$$E_x - iE_y = -f'(z) = -F(z), \tag{1}$$

$$D_x - iD_y = -\varepsilon f'(z) = -\varepsilon F(z), \tag{2}$$

where the symbol i indicates the imaginary unit and ε is the dielectric constant of the electrostrictive material. In particular, $f(z)$ and $F(z)$ have the following asymptotic representations at infinity,

$$\begin{aligned} f(z) &= iE_y^\infty z + O(z^{-1}), \quad |z| \to +\infty, \\ F(z) &= iE_y^\infty + O(z^{-2}), \quad |z| \to +\infty. \end{aligned} \tag{3}$$

Based on the revised formulation in [28], the components of Maxwell stress $(\sigma_{xx}^M, \sigma_{yy}^M, \sigma_{xy}^M)$, mechanical stress $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ and total stress (the sum of Maxwell stress and mechanical stress; $\tilde{\sigma}_{xx}, \tilde{\sigma}_{yy}, \tilde{\sigma}_{xy}$) can be expressed as

$$\begin{aligned} &\sigma_{xx}^M + \sigma_{yy}^M = 0, \\ &\sigma_{yy}^M - \sigma_{xx}^M + 2i\sigma_{xy}^M = -\varepsilon[F(z)]^2, \end{aligned} \tag{4}$$

$$\begin{aligned} &\sigma_{xx} + \sigma_{yy} = \kappa F(z)\overline{F(z)} + 4Re[\Phi(z)], \\ &\sigma_{yy} - \sigma_{xx} + 2i\sigma_{xy} = \kappa F'(z)\overline{f(z)} + 2[\bar{z}\Phi'(z) + \Psi(z)] + \varepsilon[F(z)]^2, \end{aligned} \tag{5}$$

$$\begin{aligned} &\tilde{\sigma}_{xx} + \tilde{\sigma}_{yy} = \kappa F(z)\overline{F(z)} + 4Re[\Phi(z)], \\ &\tilde{\sigma}_{yy} - \tilde{\sigma}_{xx} + 2i\tilde{\sigma}_{xy} = \kappa F'(z)\overline{f(z)} + 2[\bar{z}\Phi'(z) + \Psi(z)], \end{aligned} \tag{6}$$

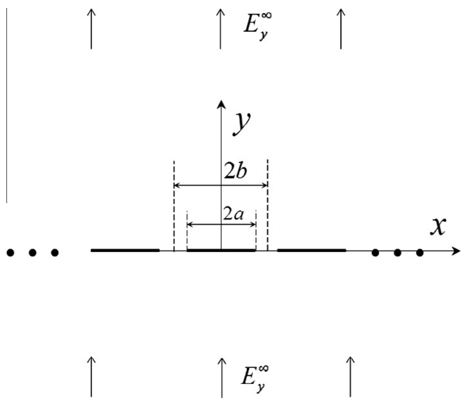

Fig. 1. Periodic cracks in an electrostrictive solid under uniform remote electric field.