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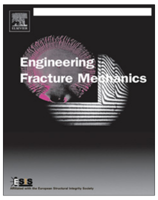

A configurational force approach to model the branching phenomenon in dynamic brittle fracture

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ABSTRACT

This publication discusses a material or configurational force approach based crack propagation scheme for dynamic fracture, in which the formulation of the material forces is derived from a Lagrangian density where inertia effects are taken into account. In dynamics, the crack driving force can generally be much larger than in the static case. In order to study the capability of the method, an algorithm based on the principle of local symmetry (PLS) is introduced into an implicit solution scheme which requires an additional iterative algorithm to seek for energy minimization. By other explicit approaches, it is not possible to study the crack bifurcation phenomenon, which is well known in dynamic fracturing.

It is observed that many micro branches evolve from the main crack in case of fast crack propagation. Thus, the energy flow into the main crack tip is divided between the main crack and the micro-branches. To introduce the micro-cracking effect to the fracture toughness, a fracture criterion as a function of the crack velocity is used in the model, in order to represent realistically the resistance of the cracked structure. In conclusion, it is shown that the proposed method based on the implicit description of energy minimization, is capable of explaining the physics behind the branching phenomenon and it offers a mesh objective solution for a structure even with a coarse mesh.

1. Introduction

The Newtonian force concept describes the relation between physical forces and deformations of particles in the physical space suitable for engineering applications. However, in fracture mechanics, additional physical and mathematical concepts due to the asymptotic behavior of stresses at the crack tip are necessary. Based on mechanics in the configurational space, a concept of generalized thermodynamic driving forces acting on imperfections of crystals such as dislocations, foreign atoms and grain boundaries is given by [1,2]. The material momentum balance equation with the divergence of Eshelby stresses can be interpreted as a representation of the negative gradient of the Lagrangian density with respect to the position of an inhomogeneity in an elastic body, e.g. crack tip, in which the inertia effects are taken into account. When inertia effects are excluded, this description coincides with the J-integral of [3] in a vectorial setting, where its tangential component with respect to the crack surface represents the variation of the configurational changes. However, for the dynamic case, more than one domain integral description is available in literature. In [4], the material force approach is developed within the context of large strain. A general application for a finite element implementation of material forces is presented by [5–9],

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Nomenclature

$\boldsymbol{X}$	reference configuration of a material point
$\mathcal{W}$	global power of the body
Π_{int}	total internal strain energy in the body
Π_{ext}	total external energy in the body
$\Pi_{\mathcal{K}}$	total kinetic energy in the body
Γ	energy dissipated by the crack surface
a	crack
$\mathcal{G}_c$	static fracture criterion
$\mathcal{G}_c^{dyn}$	dynamic fracture criterion
K^{dyn}	stress intensity factor
$\dot{a}$	explicit crack velocity
v_{lim}	velocity limiter for dynamic fracture criterion
α	fracture criterion parameter
$\boldsymbol{u}$	displacement field
$\boldsymbol{F}$	deformation gradient
$\boldsymbol{P}$	first Piola–Kirchhoff stress tensor
$\dot{\boldsymbol{\varphi}}$	velocity field
$\ddot{\boldsymbol{\varphi}}$	acceleration field
$\boldsymbol{b}_0$	body forces
t	time
ρ	density
Ψ	strain energy density
$\mathcal{K}$	kinetic energy density
$\mathcal{L}$	Lagrangian density
$\mathcal{P}$	pseudo-momentum
$\mathcal{F}$	nodal material force vector
$\mathcal{F}^{\Sigma}$	nodal volume material force vector
$\mathcal{F}^{body}$	nodal body material force vector
$\mathcal{F}^{sur}$	nodal surface material force vector
$\boldsymbol{N}$	shape function
J	J-integral in elastic fracture mechanics
$\hat{J}^D, J^D, J^{D_{\mathcal{L}}}$	variants of J-integral in dynamic elastic fracture mechanics
Λ	incremental size of the crack
$\boldsymbol{e}$	unit vector of crack kinking direction
ϕ	crack kinking angle
$\mathcal{T}$	triangular objects
$\mathcal{S}$	line objects
$\mathcal{N}$	node objects
$\boldsymbol{n}_S$	normal of a line object
$\Phi_{\mathcal{N}_{tip}}$	crack kinking direction equation
$\boldsymbol{f}^{\mathrm{r}}$	holding back force
k_P	regularization parameter
E, μ, ν, κ	material parameters
C_d	dilatational wave speed
C_r	Rayleigh wave speed.
$\nabla(\bullet)$	gradient respect to material space
$\nabla \cdot (\bullet)$	divergence respect to material space
$(\bullet)^T$	transpose of a tensor

for elasticity and by [10–15] for plasticity and viscoelasticity. Furthermore, an r-adaptive approach for a brittle fracture process in a linear elastic body is given by [16,17] using the material force approach. In addition, this formulation is discussed in the context of an explicit solution for dynamically loaded structures in [18,19].

Unfortunately, the previous applications of the material force approach to fracture mechanics are mostly restricted to quasi-static loading although a comprehensive study of various domain integrals in dynamic cases is available in literature, see [20–22], among others. Although, the framework for calculations of the material forces at large strain problems with inertia effects is given in [18,19], the applications in the literature are very limited in dynamic brittle fracture and the reliability of the method is still under discussion. Since further studies are still necessary to investigate the features of