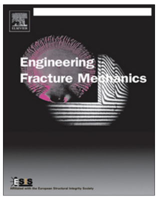
Engineering Fracture Mechanics 152 (2016) 26–57

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech



# On numerical aspects of different updating schedules for tracking fracture path in strain localization modeling

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A R T I C L E   I N F O

*Article history:*
Received 14 July 2015
Accepted 20 November 2015
Available online 7 December 2015

*Keywords:*
Cohesive zone modeling
Enhanced finite element
Fracture path
Tracking strategy
Updating schedule

## ABSTRACT

A comparative study is performed that investigates numerical features of different schedules, end-of-step vs. within iterations, for updating fracture path by employing the local and global tracking strategies. Embedded strong discontinuities within an enhanced finite element framework are used to model propagating discontinuities and fracture behavior of quasi-brittle materials. It is shown that end-of-step updating, which is a standard, can cause inaccuracies in peak strength and fracture energy for large time steps. Updating within iterations rectifies the accuracy issues, but at the expense of an increased computational cost. Both schedules yield comparable performance as the step size is refined.

## 1. Introduction

Materials subjected to certain loading conditions are predisposed to exhibit considerable deformation across narrow regions compared to the majority of the body. This phenomenon is generally termed as localized deformation. Tensile and shear fractures in brittle rocks and concrete, and shear bands in sands are examples of geomaterials in which this type of deformation is observed. Much work has been carried out in recent decades aiming to identify the criteria for onset of localization as well as the mechanics of localizing and fully localized bands.

The entire spectrum of localized deformation can roughly be classified into two families based upon their geometry, i.e., the character of the deformation in the localized region. The first group is characterized by localization bands with narrow but finite width. The displacement field throughout the body has no jumps or discontinuities though there is intense straining at the localized region. The strain field may become discontinuous. This type of discontinuity is generally termed as weak discontinuity [53], which can be observed in shear, compaction, and dilation bands in ductile materials. The second group, which is referred to as a strong discontinuity [53], includes localization surfaces with zero thickness. Localized deformations of this type have discontinuities or jumps in the displacement field at the localization surface that results in unbounded strain. Tensile fractures in rock, mortar, and concrete are examples of this type.

Modeling localized deformation is recognized to be a challenging task, partially because the governing equations of equilibrium may lose ellipticity, which results in a spurious mesh dependence. Different techniques have been developed in recent decades to surmount the associated issues. Viscous regularization (e.g. [44]), nonlocal (e.g. [33]) and gradient plasticity (e.g. [41]), and Cosserat continua (e.g. [17,19,35]) are all within a class of techniques where auxiliary information is added to the governing PDEs to ensure ellipticity and remove spurious mesh dependence of the solution. These techniques require knowledge of a characteristic length scale to complete their formulation. In addition, several elements across the thickness of

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http://dx.doi.org/10.1016/j.engfracmech.2015.11.011
0013-7944/

## Nomenclature

*Latin characters*

| | |
|---|---|
| $\mathbf{b}$ | body force |
| $\mathbf{B}$ | strain–displacement matrix |
| $c$ | prescribed elastic modulus term |
| $\mathbf{C}$ | modulus tensor |
| $d$ | prescribed displacement (mm) |
| $\mathbf{d}$ | unknown displacement vector |
| $E$ | Young's modulus (MPa) |
| $f$ | softening function |
| $f_1, f_2$ | quantities representing balance equations |
| $f_t$ | tensile strength (MPa) |
| $G$ | shear modulus (MPa) |
| $G_F$ | fracture energy (N/mm) |
| $H$ | Heaviside function |
| $\mathbf{K}$ | stiffness matrix/thermal conductivity tensor |
| $\mathbf{l}$ | surface orientation |
| max | maximum |
| $\mathbf{n}$ | normal to discontinuity surface |
| $\mathbf{N}$ | finite element shape function matrix |
| $\mathbf{q}$ | conduction flux-like vector (W/m$^2$) |
| $\mathbf{r}$ | localization residual |
| $\mathbf{R}$ | standard residual |
| $S$ | discontinuity surface |
| $t$ | time |
| $\mathbf{t}$ | traction |
| $T$ | temperature |
| tol | relative convergence tolerance |
| $\mathbf{u}$ | displacement field |
| $\bar{\mathbf{u}}$ | regular displacement |
| $\tilde{\mathbf{u}}$ | conforming displacement |
| $\mathbf{w}$ | displacement jump |
| $\tilde{w}$ | maximum of displacement jump magnitude |
| $\mathbf{x}$ | global coordinates of a material point |

*Greek characters*

| | |
|---|---|
| $\Gamma$ | material boundary |
| $\delta$ | Dirac delta distribution |
| $\epsilon$ | isotropic algorithmic conductivity |
| $\boldsymbol{\varepsilon}$ | strain field |
| $\theta$ | angle change of discontinuity line between adjacent elements |
| $\nu$ | Poisson's ratio |
| $\boldsymbol{\nu}$ | outward normal to material boundary |
| $\boldsymbol{\sigma}$ | stress field |
| $\Omega$ | material domain |

*Sub/superscripts, abbreviations, and special functions*

| | |
|---|---|
| $+$ | active domain |
| $-$ | inactive domain |
| $*$ | prescribed value |
| $c$ | critical |
| conf | conforming part |
| conv | converged |
| CST | constant stain triangular |
| $dd$ | derivative of standard residual with respect to displacement |
| $dw$ | derivative of standard residual with respect to jump |
| $e$ | (subscript) element |
| $e$ | (superscript) elastic |
| $f^h$ | arbitrary smooth function |