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A smooth unloading–reloading approach for the nonlinear finite element analysis of quasi-brittle materials

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ABSTRACT

This paper presents a new method for improving the robustness and convergence characteristics of a finite element damage model for quasi-brittle materials. In this method, a smooth unloading–reloading function (SUR) is employed to compute an approximate tangent matrix in a Newton type solution procedure. A new method is also presented for predicting a converged value of the damage parameter. The performance of the new methods are assessed using a range of idealised quasi-brittle specimens. Results show that the new SUR approaches are robust and result in considerable savings relative to solutions obtained with a secant unloading–reloading function.

1. Introduction

Micro-cracking is a feature of quasi-brittle (QB) materials loaded beyond their elastic limit and is the primary cause of stiffness and strength degradation in materials such as concrete and rock. Laboratory samples of quasi-brittle material frequently exhibit a post-peak softening response when loaded in tension or unconfined compression. This macro-scopic softening behaviour is sometimes referred to as material softening although it is recognised that this is a structural phenomenon, resulting from the micro-cracking, rather than a fundamental response of the material [1–4].

Softening behaviour has presented researchers with two related computational challenges; namely, how to (i) obtain mesh-objective predictions and (ii) find minimum energy converged solutions in an efficient manner. Mathematically, these issues are a consequence of the loss of ellipticity of the governing partial differential equations [5], when a certain degree of damage is exceeded, and are characterised by the associated stiffness matrix becoming non-positive definite.

The first of the above challenges can be dealt with, at least to first order accuracy, by using the crack-band model of Bažant and Oh [6]. More refined means of resolving the mesh-sensitivity problems include the use of integral [7,8] and differential [9–12] non-local models. However, resolving the mesh sensitivity issue does not resolve of all the stability and convergence issues associated with modelling QB materials.

The nonlinear equations resulting from the finite element simulation of QB structures are frequently solved using incremental-iterative solution schemes based on Newton–Raphson (NR) algorithms [5,13]. It is the poor convergence properties of these solution schemes, when solving problems involving QB materials, which so frequently cause frustration to finite element analysts. It is this issue that provides the motivation for the work of this paper.

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Nomenclature

a_k	constant used in computing Softening curve constants
a_p	softening function constant
a_t	softening function constant
c_1	softening function constant
$\mathbf{D_0}$	elastic stiffness matrix
$\mathbf{D}_{tan}$	constitutive tangent matrix
E	Young's modulus
E_c	Young's modulus of concrete
E_s	Young's modulus of steel
f_s	target softening function
f_t	tensile strength
G_f	fracture energy
It	iteration number
m	softening function constant
r_0	effective end of the softening curve
r_{eff}	effective damage evolution parameter
r_k	damage evolution parameter at the peak of the uniaxial stress curve
r_p	damage evolution parameter
r_{pp}	predictive damage evolution parameter
r_t	effective damage strength parameter $= f_t/\sqrt{E}$
r_ε	limiting damage evolution parameter ratio
β	softening curve constant
Δr_p	iterative change in damage evolution parameter
$\boldsymbol{\varepsilon}$	strain tensor
ε_0	strain at the effective end of softening curve
ε_t	tensile strain measure
η	target softening curve constant
η_k	normalised damage evolution parameter
ν	target softening curve constant
υ	Poisson's ratio
$\boldsymbol{\sigma}$	stress tensor
$\boldsymbol{\sigma_0}$	effective stress
σ_k	stress to which the SUR function is asymptotic
σ_p	smooth unloading–loading function
Ψ_d	tolerance value for displacement norm
Ψ_f	tolerance value for out of balance force norm
ω	basic damage variable ($\omega \in [0, 1]$)
ω_p	SUR damage parameter
ω_{pf}	damage parameter controlling linear part of the SUR function

Many approaches have been made to improve the efficiency of these NR procedures and to improve their robustness [5,13–17]. These techniques include replacing standard Newton schemes with Quasi-Newton approaches [13,14,17] and accelerated NR methods, amongst which line-search algorithms are one of the most effective techniques [5,13,15,16].

When the global response of a structure softens and exhibits 'snap-back' behaviour, arc-length procedures can allow the complete equilibrium path to be traced [5,13,17–21]. The constraints provided by the arc-length methods can also help to improve the overall stability of a solution as well as allowing solutions to be obtained when local snap-backs occur. Another approach for tracing global snap-back is Ladevese's LATIN method [22], which has undergone significant development in recent years [23,24].

None of the aforementioned algorithms are completely robust, nor do they fully resolve all the stability and convergence difficulties encountered when analysing QB structures. These on-going difficulties have undoubtedly been behind the development of solution algorithms that avoid multiple iterations. These methods include the 'implicit–explicit' approach of Oliver et al. [25,26], in which a projected state variable (e.g. a damage parameter) is used to determine a predicted consistent tangent matrix that is exact for the current increment but for which a correction is made in the subsequent stress-recovery phase. An alternative non-iterative method, called the 'Sequentially Linear Approach (SLA)', was proposed by Rots [27]. This method uses a 'saw-tooth' function to replace the post-peak softening function. Rots and Invernizzi [28] and Rots et al. [29] later improved the SLA to achieve mesh independent behaviour. The SLA has also been extended to allow the simulation of non-proportional loading [30,31], as well as being applied to the analysis of concrete beams which fail in shear [32].