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The fracture mechanics of cantilever beams with an embedded sharp crack under end force loading

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A R T I C L E I N F O

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A B S T R A C T

Motivated by the need to develop model and non-model based methods for damage and crack detection in components and structures, this paper aims at establishing the near-tip mechanics of cantilever beams containing a fully embedded, through thickness sharp crack and subjected to an end force. Finite element (FE) models of the cracked beams were established with the aid of a specialized 2-D adaptive meshing algorithm. Beam geometries with a crack placed at various depths and locations along the beam axis and at various orientations have been modeled. Linear elastic and isotropic conditions were assumed throughout the homogeneous beam domain. Broad parametric studies were conducted to study the effects of crack length, crack orientation and crack location on the fracture characteristics dominating both the left and right crack tips. As such extensive results are reported for the near tip energy release rates and the associated Mode I and Mode II stress intensity factors and mode mixity.

The study suggests that the near-tip conditions for both the left and right crack tips in systems with non-horizontal cracks are dominated by mixed mode conditions. Physically inadmissible crack surface interpenetrations are predicted associated with negative Mode I stress intensity factor component for at least one of the two crack tip regions for all incline cracks. The extent of crack surface interpenetration is shown to depend on the crack plane orientation relative to the beam axis.

For beams containing a horizontal crack, i.e., cracks aligned with the beam axis, the simulation results suggests that such cracks are dominated by Mode II conditions at both crack tips regardless of its length and crack location in the beam. The findings of this study along with other related results regarding the deformation of a beam with an embedded horizontal crack form the foundation for the development of analytical models capable of capturing the overall deformation as well as the near-tip fracture characteristics of such cracked structures. In addition, the findings can assist in furthering our understanding of delamination processes in laminate systems and in developing model and non-model methods for damage and crack detection.

1. Introduction

Health monitoring of mechanical components, systems and structures has received renewed attention in recent years primarily due to the aging of the infrastructure and increased use by rapid population growth. According to Doyle et al. [1],

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Nomenclature

x, y	Cartesian coordinates
x_C, y_C	x, y coordinates of the sharp crack center
L	beam length
L_c	characteristic length
h	beam height
P	load
P_c	characteristic load
E	elastic modulus
E_c	characteristic modulus
v	Poisson's ratio
$2a$, l	crack length
$2\Delta a$, Δl	Virtual Crack Extension (VCE)
θ	crack orientation angle with respect to x-axis, counter-clockwise is positive
$\Im$	energy release rate
$\Im_c$	characteristic energy release rate
K_I	Mode I stress intensity factor (SIF)
K_{II}	Mode II stress intensity factor (SIF)
K_{ch}	characteristic SIF, as normalization factor
Ψ	mixed mode phase angle, also known as mode mixity
$\{u\}$	nodal displacement vector
N_c	number of elements participating in the implementation of the VCE method
i	elements counting index used in the VCE method
$[k_i^c]_l$	element stiffness obtained for a meshing containing a crack of length l
$[k_i^c]_{l+\Delta l}$	element stiffness obtained for a meshing containing a crack of length $l + \Delta l$

the population of the Unite States has grown from 90 million in 1900 to over 300 million in 2000. As stated in [1], over the same 100-year period, the above population growth was accompanied by a rapid expansion of the civil infrastructure building over 68,000 dams, 600,000 bridges, 530,000 miles of sewer pipes and about 3.6 million miles of surface roads. In recent years, frequent and costly infrastructure failures, such as water main breaks in major metropolitan areas, bridge aging and failures, and building collapses, have provided the impetus for the development of advance diagnostic and structural health monitoring tools.

In the past several decades, diagnosis and identification of damage in components and structures has been a field of challenging research and numerous related technical contributions have been reported. Vibration-based methods [2,3] utilizing the systems' frequency [4–7] and modal response [8,9] have been shown capable of predicting the presence of damage manifested as a local reduction in the component's structural stiffness. While these methods were shown to reliably predict the location of the damage along the beam axis, they have limited sensitivity to assess the type of damage, its extent and sub-surface location. In addition, the methods do not possess the sensitivity to differentiate between structural stiffness degradation due to modulus reduction caused for example by progressive corrosion or structural degradation caused by geometric flaws such as voids and cracks.

To address these persisting challenges, fracture mechanics concepts such as the J-integral [10] associated with the presence of sharp cracks were employed in damage detection by solving the forward problem associated with a component of given geometry and material composition containing a sharp crack of known size, orientation and crack center location. For example, Ioakimidis [11] developed a general fracture mechanics based method for nondestructive testing. In his study, he was able to locate the presence of a crack of known shape embedded in an isotropic elastic medium utilizing estimates of the path-independent J-integral. Lei [12–14] studied semi-elliptical surface cracks in plates under tension and bending utilizing the concept of J-integral and Finite Element Method (FEM).

In studying the fracture mechanics of a cracked system, the FEM has been used extensively, especially after the 1960s when progressively increased computational power became more readily available for computational research. For example, utilizing fundamental fracture mechanics concepts developed by Rice [10], Rice et al. [15] developed a finite element based stiffness derivative method in extracting from a known finite element solution the associated elastic energy release rate for a crack under Mode I conditions. Motivated by the need to understand the characteristics of fracture at bimaterial interfaces, a phenomenon prevalent in heterogeneous fiber reinforced composites and composite laminates, Charalambides et al. [16] and Matos et al. [17] extended Park's stiffness derivative method to interface cracks under mixed mode conditions. In the latter studies, they were able to establish the relative contributions of Mode I and Mode II to the mechanics of an otherwise mixed mode crack. Zhang and Charalambides [18] further improved the above methods to include interface cracks bounded by heterogeneous orthotropic media. Skrinar [19] utilized FEM to model a beam with an arbitrary number of transverse cracks. The model was then simplified such that each crack was replaced by a corresponding linear rotational spring,