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The Griffith Medal Lecture: The fracture mechanics of soft solids

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ABSTRACT

A review is given of describing the fracture of soft solids using the notion of energy release rate as proposed by Griffith. Soft solids have low modulus and, sometimes, low yield stresses which result in self-blunting when cracked specimens are loaded laterally. The resulting blunt cracks bring into play the requirement that some critical stress must be achieved as well as the energy release rate. This two criteria analysis is explored and conditions for sharp crack failure, blunt crack failure with elevated apparent toughness and ductile tearing when the critical stress is not achieved are described.

1. Introduction

I was honoured to be awarded the Griffith Medal by ESIS and to give the Medal Lecture at ECF20 in Trondheim. I chose to talk about applying energy based fracture mechanics as pioneered by Griffith, to the study of fracture in soft solids. This is a relatively new field but is becoming increasingly important in the characterisation of biological materials and foods.

I believe the energy based approach to such problems to be profound and the two papers written by Griffith [e.g. 1] provide the basis on which an understanding can be based. On a personal note I have always felt an affinity with Alan Griffith since we share a common birthdate, 13th June, were both brought up in Liverpool and our first job was at the Royal Aircraft Establishment in Farnborough. Both of his only two fracture papers were written while he was at Farnborough and he introduced the concept of Energy Release Rate, (G, in honour of Griffith) as the crack driving force which will be used throughout here. He also addressed the necessity of achieving a sufficient cohesive stress at the crack tip to cause fracture in addition to the necessary energy release via G. He observed that for sharp cracks the stress concentration was generally sufficient to achieve this and that $G \geqslant G_c$, some critical property value, was a sufficient single fracture criterion and can be used for brittle fracture where cracks remain sharp. The issue for soft solids is that the second criterion is not necessarily achieved because of crack blunting and we need more than a single criterion.

This issue has been recognised in the study of the fracture of rubber in the 1960s. Work at the Natural Rubber Research Association by a very talented group including Rivlin, Thomas, Gent, Lake and Lindley [e.g. 2,3] extended the G concept to finite strains and recognised that crack blunting was an important factor. Indeed they defined a Tearing Energy when the material failed with the crack blunted by the large strains and measured values of 20 kJ m^{-2} for natural rubber. They devised tests in which razor cutting was used and G calculated from the cutting force and observed a value of 0.5 kJ m^{-2}. It was noted

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Nomenclature

a	crack length
b	sheet thickness
c	$EG_c/2\pi\sigma_c^2$
$\bar{c}$	critical value of c for fracture
d	wire diameter (in wire cutting)
E	Young's modulus
F_c, F_t	forces in the cutting and transverse directions
G	strain energy release rate
G_c	critical strain energy release rate
G_b	strain energy release rate for a blunt crack
h	cut thickness (in orthogonal cutting)
h_c	chip thickness (in orthogonal cutting)
K	stress intensity factor
K_c	critical stress intensity factor
k	function of yield strain (see Eq. (10))
x	distance from notch root
α	rake angle (of a cutting tool)
e_y	yield strain
ϕ	shear plane angle
μ	coefficient of friction (between the wire and the soft solid)
ρ	crack tip radius
ρ_c	critical crack tip radius (below which $K = K_c$)
ρ_e	elastic contribution to crack tip radius
ρ_p	plastic contribution to crack tip radius
ρ_b	blunted crack tip radius
σ	stress
σ_c	stress at fracture
σ_y	yield stress

that this was the true toughness and was the controlling factor in, for example, the failure of tyres. The work is notable for skilful experimental work and the pursuit of sound physics. What is described here is done in, what I hope, is the same spirit.

Also in the 1960s George Irwin [4] addressed the issue of blunt cracks in the context of crack sharpness in fracture toughness testing. He considered the stress at the tip of a crack of finite radius ρ and noted that the stress intensity factor ($K^2 = EG$) to achieve a critical stress is given by

$$\frac{K}{K_c} = \sqrt{\frac{\rho}{\rho_c}}, \quad \rho \geqslant \rho_c \tag{1}$$

where ρ_c is a critical radius below which $K = K_c$ the sharp crack values and;

$$\frac{K}{K_c} = 1, \qquad \rho \leqslant \rho_c \tag{2}$$

Or, in terms of G,

$$\frac{G}{G_c} = \frac{\rho}{\rho_c} \text{ and } \frac{G}{G_c} = 1 \tag{3}$$

In this case ρ is the initial radius of the crack tip and does not arise from self-blunting. The notion of ρ_c is important in all fracture mechanics testing standards.

2. Fracture in soft solids

A "Soft" solid is defined here as a material which has a low elastic modulus, E, and, in some cases, a low yield stress. The top end of the scale are soft polymers with E = 1 GPa and σ_y = 10 MPa, i.e. a yield strain e_y = 0.01, down to starch gels with E = 0.1 MPa, σ_y = 0.05 MPa and e_y = 0.5. Many very soft materials have no real yield stress and are elastic, often with some visco-elasticity. An example of what occurs when stretching a soft polymer (PE) is shown in Fig. 1 in which a sharp crack self-blunts because of the large strains. Of course, initially blunt notches continue to blunt further on loading.

A useful model of these effects is the linear elastic analysis of a blunt crack of length, a, and a tip radius ρ subjected to a stress σ. The stress at a distance r from the crack tip is given by Inglis [5],

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