Engineering Fracture Mechanics 149 (2015) 351–367

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Mixed mode partitioning of beam-like geometries: A damage dependent solution

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ARTICLE INFO

Article history:
Received 29 December 2014
Received in revised form 4 June 2015
Accepted 17 June 2015
Available online 2 July 2015

Keywords:
Cohesive zone model
Mixed mode fracture
Partitioning
Fracture test methods

ABSTRACT

The current work studies the fracture mode partition in beam-like geometries as a function of cohesive properties. It is observed that the mode mixity exhibits a unique dependence on the cohesive zone length scale, where the lower and upper bounds are given by the local and global partitioning. Based on this observed unique dependency, a new semi-analytical cohesive analysis (SACA) is proposed for partitioning. This partitioning method is applied to previously conflicting experimental data in the literature, and physically consistent results are obtained in each case, suggesting that this novel SACA method can be used to obtain accurate mixed mode partitioning values.

1. Introduction

It has been widely documented that the toughness of adhesive joints and composite laminates can vary considerably depending on the mode of loading [1–4]. In many of these cases, the mode I energy release rate (ERR), G_{IC}, is shown to be the most critical, and hence is a conservative value when used in design. However, Dillard et al. [3] highlighted a number of cases where the lowest measured ERR, G_C, occurs at a certain mixed mode I/II loading. To eliminate the risk of non-conservative design, and also avoid over design, it is important to characterise joint toughness over a full range of mixed mode loadings ranging from pure mode I (tensile) to pure mode II (in-plane shear). Test methods for fracture characterisation of the pure modes I and II have been well established [5,6]. However, while a number of options exist for mixed mode I/II testing [7–9], it is an area of considerable contentiousness and uncertainty, in particular regarding the mode decomposition of asymmetric specimens.

Beam-like geometries as shown in Fig. 1 have been the main basis for the development of analytical mixed mode partitioning theories, and will be the focus of the current study. The mode mixity (G_{II}/G) is defined here as the ratio of the energy release rate in mode II (G_{II}) to the total energy release rate (G), where $G = G_I + G_{II}$. Williams [10] proposed a fully analytical approach for decomposing ERRs into mode I and mode II components based on classical beam theory, without considering the details of local stress and strain distributions at the crack tip. This approach is often referred to as the global approach. Yin and Wang [11] and Suo and Hutchinson [12] proposed a solution based on the decomposition of stress intensity factors (SIFs) by considering local conditions at the crack tip. Thus, in contrast to the Williams [10] global partitioning approach, this approach assumes the presence of a K dominant region and is commonly referred to as the local partitioning approach. The

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http://dx.doi.org/10.1016/j.engfracmech.2015.06.061

Nomenclature

Symbol	Description
G_I	mode energy release rate
G_{II}	mode II energy release rate
G_{IC}	mode I critical energy release rate
G_{IIC}	mode II critical energy release rate
G	energy release rate
h_1	upper beam height
h_2	lower beam height
γ	beam height ratio
M_1	bending moment applied to upper beam
k	applied bending moment ratio
M_I	bending moment producing pure mode I
M_{II}	bending moment producing pure mode II
B	specimen thickness
K_I	mode I stress intensity factor
K_{II}	mode II stress intensity factor
ω	linearly averaged mixed mode parameter
$J_{\Gamma_{ext}}$	J integral evaluated externally around crack tip and FPZ
$J_{\Gamma_{coh}}$	J integral evaluated over cohesive surfaces
$J_{\Gamma_{tip}}$	J integral evaluated over small contour surrounding crack tip
J_I	mode I component of mode decomposed J integral
J_{II}	mode II component of mode decomposed J integral
δ_N	normal opening displacement
δ_S	shear opening displacement
t_N	normal traction
t_S	shear traction
t_{NC}	critical normal traction
t_{SC}	critical shear traction
D	damage variable
k_{xx}	initial penalty stiffness in the shear direction
k_{yy}	initial penalty stiffness in the normal direction
l_{cz}	cohesive zone length
a_c	smallest characteristic dimension
l_{nd}	normalised cohesive zone length
f	local–global linear averaging parameter
M	Scale factor
l_{I_cz}	pure mode I cohesive zone length
l_{II_cz}	pure mode II cohesive zone length
l'_{I_cz}	mode I contribution to cohesive zone length in mixed mode
l'_{II_cz}	mode II contribution to cohesive zone length in mixed mode

predicted mode partition from both the local and global approaches are independent of material properties and depend only on γ and k, where γ is the ratio of beam heights and k is the ratio of applied moments as defined in Fig. 1. The predicted partitions from both methods agree *only* when the crack is located centrally ($h_1 = h_2$), as shown in Fig. 2(a) where the predicted mixed mode partitions (G_{II}/G) from the local and global approaches are plotted as a function of the applied moment ratio k for symmetric specimens. In cases where $h_1 \neq h_2$, the differences between the local and global approaches are significant, as is evident in Fig. 2(b) where the predicted mixed mode partition (G_{II}/G) from both approaches are plotted as a function of the beam height ratio γ for $k = 0$.

Numerous experimental studies have investigated the accuracy of both the local and global approaches. Hashemi et al. [1] carried asymmetric tests on carbon epoxy composite and applied both local and global partitioning. It was found that the global approach produced a more physical failure locus and it was proposed that this was likely due the fracture process zone being larger than the relatively small K dominant region (often $\leqslant 1\%$ of smallest beam thickness [13]). Davidson et al. [14] also carried out asymmetric tests on carbon epoxy composite and compared the resulting failure loci, obtained via local and global partitioning, to the already measured *true* failure locus obtained using symmetric specimens. It was found that neither the local nor global approach predicted the mode partition adequately. Ducept et al. [15] undertook a similar study on glass reinforced epoxy composite and found that the local partitioning accurately predicted the symmetrically measured failure locus.