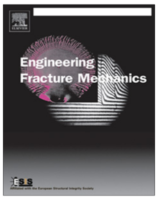
Engineering Fracture Mechanics 135 (2015) 1–16

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech



Stochastic discrete meso-scale simulations of concrete fracture: Comparison to experimental data

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ARTICLE INFO

Article history:
Received 4 August 2014
Received in revised form 4 January 2015
Accepted 5 January 2015
Available online 17 January 2015

Keywords:
Discrete model
Lattice-particle models
Energy dissipation
Statistical methods
Size effect
Fracture testing
Notch depth effect
Concrete structures

ABSTRACT

The paper presents a discrete meso-scale model for fracture of concrete taking into account random spatial variability of material parameters. Beams of various sizes, with notches of various depths, are simulated numerically to study the combination of energetic and statistical size effects. A substantial part of material randomness is shown to be caused by random locations of the largest aggregates. Further randomness, due to random fluctuations of material parameters, is considered and an effect of introducing a spatially auto-correlated random field is analyzed. The results of the simulations are compared with recently published experimental data on concrete beams in three-point bending. The differences in the role of randomness in beams of various sizes, with different notch depths, are demonstrated, and differences in energy dissipation are discussed.

1. Introduction

The reliability of reinforced concrete structural members is crucial for modern engineering structures. When evaluating the reliability of concrete structures, the fracturing of concrete is one of the crucial phenomena that needs to be included in the analysis. Many features responsible for variability in resistence of concrete can be named. The irregular inner structure of concrete, characterized by random spatial arrangement of grains of various sizes and spatial variability of material properties calls for theoretical model that is able to account for these features. With the help of such a model, the behavior of concrete structures can be studied, understood and predicted, which is needed for design and assessment of engineering structures. The model parameters are usually obtained from small size laboratory specimens, but applying these parameters to large structures poses difficulties. One of them is the spatial randomness of material properties, which is usually insignificant and is ignored for the mean response of small specimens, but becomes significant in large structures.

The need to understand concrete fracture has resulted in the development of complex numerical models that can predict the strength and softening. It is generally agreed that the fracture process in concrete and other quasibrittle materials (ceramics, ice, etc.) is characterized by the gradual release of stress, or softening, within the fracture process zone (FPZ)

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http://dx.doi.org/10.1016/j.engfracmech.2015.01.004

Nomenclature

D	beam depth
S	beam span
b	beam thickness
a_0	notch depth
α_0	relative notch depth
$\widetilde{E}$	macroscopic elastic modulus
$\widetilde{\nu}$	macroscopic Poisson's ratio
E_a	elastic modulus of grains
E_c	elastic modulus of matrix
α	parameter controlling Poisson's ratio
f_t	tensile strength
f_s	shear strength
G_t	fracture energy in tension
G_s	fracture energy in shear
P	loading force
A	area under load–CMOD curve
γ	damage variable
σ_{n}	nominal stress
g	dissipated energy
l_c	correlation length
$\boldsymbol{H}$	random field
$\widehat{\boldsymbol{H}}$	Gaussian random field
$\boldsymbol{x}$	spatial coordinate
H	random variable
F_H	cumulative distribution function
Φ	cdf of the standard Gaussian distribution
$\boldsymbol{C}$	covariance matrix
ξ	standard Gaussian variable
ρ	correlation
λ	eigenvalue of covariance matrix
ψ	eigenvector of covariance matrix

ahead of the macroscopic crack tip. This gradual softening is understood to be a consequence of concrete heterogeneity. This understanding has led to attempts to include the heterogeneity directly in the material model. Although heterogeneity can be simulated using continuous material description [6], the meso-level simulation of concrete fracture is often performed using discrete models. The simplest and least phenomenological of them are the classical lattice models [14,21,41,26,37,42,38,30], which feature elasto-brittle lattice elements and lattice geometry independent of the material heterogeneity. However, such models require a high resolution, even within the dominant heterogeneities, making them computationally expensive and hence suitable for small specimens only. A reasonable compromise seems to be using a less dense lattice with each node corresponding to one dominant mineral aggregate. Such models may have only translational degrees of freedom and axial connections (i.e., central forces) between grains, as in a truss [4,28,29]. An early model of this kind, efficient enough even for the 1970s computers, was Burton and Dougill's [5] network model, which is recently being emulated by the "peridynamic" models despite the serious inherent limitations of the central force lattice.

A major improvement, making the discrete model much more realistic, was the two-dimensional aggregate interface model [47] in which the particle rotations and interparticle shear interactions were taken into account. This approach was greatly improved and generalized to three dimensions in [8] and recently further refined in the works of Cusatis et al. [10,11]. It is important that his model, called the lattice-particle model, can take into account the three-dimensional rotations of particles or grains, which cause shearing and bending in particle contacts. This can be captured by constitutive law formulated in terms of interface stress and strain vectors, in a way similar to the microplane model. It is a significant advantage that the vectorial constitutive law can directly reflect not only crack opening but also frictional and dilatant slip on plane of distinct orientations. The recent extensions of the lattice-particle model include effects such as chloride diffusion [40,1], fiber reinforcement [31,32] or fatigue [19,13]. A comparison of failure events in the discrete model to acoustic emission measurements during compression test can be found in [27].

The modeling of the fracture process is further complicated by random fluctuations of mechanical properties in concrete. These fluctuations have several sources, among which the randomness in the concrete constituents themselves (material properties, geometric properties), the process by which the constituents are mixed (aggregate locations, non-homogeneous distribution of water, cement, finer aggregates and additives), and non-uniform drying are the most significant. To identify