Engineering Fracture Mechanics 135 (2015) 232–244

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Technical Note

Double cantilever beam model for functionally graded materials based on two-dimensional theory of elasticity

P. Chu [a], X.-F. Li [a,b,*], Z.-G. Wang [c], K.Y. Lee [b]

[a] School of Civil Engineering, Central South University, Changsha 410075, PR China
[b] State Key Laboratory of Structural Analysis for Industrial Equipment and Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, PR China
[c] College of Resources & Planning Sciences, Jishou University, ZhangJiajie 427000, PR China

ARTICLE INFO

Article history:
Received 19 December 2014
Received in revised form 7 January 2015
Accepted 11 January 2015
Available online 19 January 2015

Keywords:
Functionally graded material
Double cantilever beam
Exponential gradient
Energy release rate
Stress intensity factor

ABSTRACT

This paper extends the double cantilever beam (DCB) model to functionally graded materials (FGMs) based on two-dimensional theory of elasticity. A theoretical approach is proposed to determine strain energy release rate of a crack in a layered FGM with exponential gradient perpendicular to the crack plane. Elasticity solutions of a functionally graded cantilever subjected to a concentrated force and a bending moment are derived for thickness-wise varying Young's modulus. Using the derived elasticity solutions, the energy release rate and stress intensity factor for an edge crack and a central crack parallel to the surface are obtained. Numerical results are given to show the influence of gradient index on fracture parameters.

1. Introduction

Functionally graded materials (FGMs) refer to the materials with continuously varying material properties and have certain advantages such as eliminating deformation mismatch, enhancing bonding strength and reducing crack driving force over two bonded dissimilar materials with distinct interface. For instance, a homogeneous elastic layer of ceramic material may be bonded to the surface of a metallic structure and acts as a thermal barrier in high-temperature environment. However, a distinct interface of ceramic/metal bi-material is one of potential crack initiation sites thanks to mismatch of material properties and deformation. Therefore, the design of FGMs permits the material properties to gradually transit from one material to another material so that mismatch of material properties can be effectively avoided via gradually changing the volume fraction of the constituents. Consequently, the performance of FGMs is much improved.

Crack problems in FGMs are an important subject of researches in accessing structural integrity and safety. Great progress has been made in theoretical analyses on crack problems of FGMs. In most of the published papers, stress intensity factor (SIF) and energy release rate (ERR) are only numerically determined through various analytical and numerical approaches including the integral equation method, see e.g. [1–7], finite element method [8], boundary integral equation method [9], asymptotic method [10], interaction energy integral method [11], etc. Although these methods provide effective approaches for acquiring SIFs of the crack tip in FGMs, the results are all numerical. No one can give an analytic closed-form solution for SIFs.

* Corresponding author at: School of Civil Engineering, Central South University, Changsha 410075, PR China. Tel.: +86 731 8816 7070; fax: +86 731 8557 1736.

E-mail address: xfli@csu.edu.cn (X.-F. Li).

http://dx.doi.org/10.1016/j.engfracmech.2015.01.008
0013-7944/

Double cantilever beam (DCB) is a simple model to determine SIFs of a crack in a thin elastic layer [12] and is also taken as a typical experimental approach to measure an interlaminar fracture resistance in homogeneous materials [13–16]. In particular, this approach allows one to analytically obtain SIFs in closed form and provides an enough accurate approximate result as compared to the exact ones [17,18]. Davidson [19] established a DCB model to calculate ERR for Mode I crack and analyzed the effects of delamination front curvature on the fracture toughness. Nairn [20] gave some experimental methods to correct the error between the apparent toughness and true toughness with special emphasis on the residual stress effects in DCB. Guo et al. [21] pointed out that the ERR is independent of crack length, but is a function of residual stress level for DCB specimens. This effects of residual stress is expected to be small but cannot be ignored. A tapered DCB model has been considered in which the effects of the root rotation and residual stress are analyzed through simple shear-corrected beam theory [22]. Using the Timoshenko beam theory, some researchers obtained more accurate numerical result of stress intensity factors for a DCB with its root as a linear spring [23–25]. The classical DCB model has been used to micro or nano beams to account for the size effect [26,27]. An asymmetric DCB model has been formulated to assess the mixed-mode interlaminar fracture toughness of composite laminates based on various theories including the nonlinear large deflection theory [28], the beam theory along with finite element method [29], the Timoshenko beam theory [30]. For an asymmetric DCB consisting of laminated composite, the measurement of the interlaminar fracture toughness has been suggested by means of the ERR via the formula of the elastic beam theory [31].

In this paper, we study the DCB model in FGMs in the framework of the two-dimensional theory of elasticity. An elasticity solution of a functionally graded cantilever (FGC) subjected to a concentrated force and a bending moment is derived by means of the Airy stress function method. A DCB model for FGMs is established and ERRs and SIFs are derived for an edge-cracked and a centrally-cracked elastic layer of FGMs. The influence of the gradient index on fracture parameters is discussed.

2. Basic equations

Consider a layered FGM with a crack, which terminates at the edge or is embedded in the FGM, as shown in Fig. 1(a and b). Young's modulus of the FGM layer varies continuously in the thickness direction or/and in the length direction. For convenience of later analysis, Young's modulus takes the following exponential form

$$E(x,y) = E_0 e^{\lambda_1 x + \lambda_2 y} \tag{1}$$

where E_0 is a reference value of Young's modulus of the FGM at the origin of the Cartesian coordinate system $(x=0, y=0)$, λ_1 and λ_2 refer to gradient indices. If $\lambda_1 = 0$ and $\lambda_2 = 0$, we have $E(x,y) = E_0$, which corresponds to a homogeneous isotropic elastic material. For most materials, Poisson's ratio has a very slight variation, and in the present paper it is assumed to be a constant, v.

In order to give an adequate description of the problem, we invoke the two-dimensional theory of elasticity, rather than one-dimensional beam theory. For this purpose, without consideration of body forces, the normal and shear stress components, $\sigma_x,\ \sigma_y$, and τ_{xy}, satisfy equations of equilibrium

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \tag{2}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0. \tag{3}$$

For a linearly elastic isotropic medium made of FGMs, the constitutive equations read

$$\varepsilon_x = \frac{1}{E(x,y)}(\sigma_x - v\sigma_y), \tag{4}$$

$$\varepsilon_y = \frac{1}{E(x,y)}(\sigma_y - v\sigma_x), \tag{5}$$

$$\gamma_{xy} = \frac{2(1+v)}{E(x,y)}\tau_{xy}, \tag{6}$$

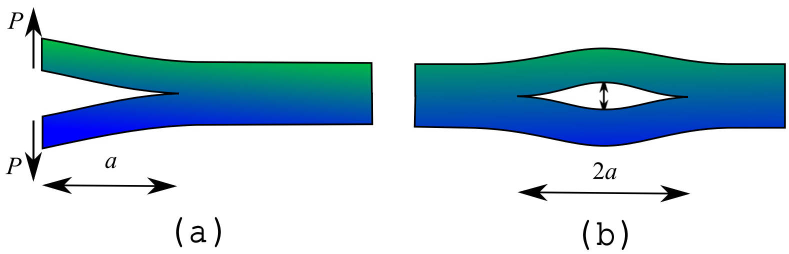

Fig. 1. Schematic of a cracked layer of FGM, (a) an edge crack parallel to the surface and (b) a central crack parallel to the surface.