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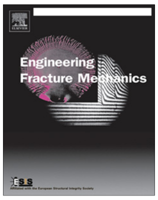

An efficient and accurate method for computation of energy release rates in beam structures with longitudinal cracks

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ABSTRACT

This paper proposes a novel, efficient, and accurate framework for fracture analysis of beam structures with longitudinal cracks. The three-dimensional local stress field is determined using a high-fidelity beam model incorporating a finite element based cross section analysis tool. The Virtual Crack Closure Technique is used for computation of strain energy release rates. The devised framework was employed for analysis of cracks in beams with different cross section geometries. The results show that the accuracy of the proposed method is comparable to that of conventional three-dimensional solid finite element models while using only a fraction of the computation time.

1. Introduction

This work is motivated by the challenges associated with the analysis of cracks in wind turbine rotor blades. The structural analysis of blades is typically performed in a finite element context. For example, Overgaard and Lund [16], Overgaard et al. [17,18] presented a solid and shell finite element model for simulating the collapse of a wind turbine blade under static loading. The fracture analysis is based on the cohesive element approach (Barenblatt [2]). More recently, Eder and Bitsche [9] presented a similar modeling approach using the Virtual Crack Closure Technique (VCCT) for the analysis of cracks in trailing edge adhesive joints of a wind turbine rotor blade. The present paper introduces a novel modeling approach combining a finite element based cross section analysis tool and VCCT.

The VCCT is a well established method for the computation of the energy release rate (ERR) based on results from finite element analysis (Rybicki and Kanninen [20], Xie and Waas [22], and Krueger [12] and references therein). The VCCT is computationally efficient and provides the modal contributions to the total ERR, where the latter is crucial for mixed mode fracture analysis. This technique is based on linear elastic fracture mechanics and on the assumption that the energy released during crack propagation equals the work required to close the crack back to its original position. Based on this assumption, the ERR is computed from the nodal forces at the crack tip and relative nodal displacements behind the crack tip. The finite element models providing the required nodal forces and nodal displacements are typically based on plane stress or strain, shell, or solid finite elements (see Krueger [12] for an extensive review on the topic). As a relatively fine mesh must be used in the area surrounding the crack, three-dimensional models of this kind are often computationally expensive. This is especially true if the location, orientation and size of the crack is not known *a priori*, and a large number of model configurations must be analyzed.

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Nomenclature

a	element length next to the crack tip
a_1, a_2	element length behind and in front of crack tip, respectively
c	element width at crack front
E_α	elastic modulus of material α
$\hat{\mathbf{f}}, \hat{\mathbf{f}}_e$	global and element load vector for beam finite element model
$\overline{\mathbf{f}}, \overline{\mathbf{f}}_e$	global and element load vector for 3D solid finite element model
$\mathbf{f}, \mathbf{f}_e$	global and element load vector for cross section finite element model
$\mathbf{F}_s$	cross section compliance matrix
$G_{I,II,III}$	Mode-I, II, and III energy release rates
H	height of the square cross section
K_{III}	Mode-III stress intensity factor
$\hat{\mathbf{K}}, \hat{\mathbf{K}}_e$	global and element stiffness matrix for beam finite element model
$\overline{\mathbf{K}}, \overline{\mathbf{K}}_e$	global and element stiffness matrix for 3D solid finite element model
$\mathbf{K}, \mathbf{K}_e$	global and element stiffness matrix for cross section finite element model
$\mathbf{K}_s$	cross section stiffness matrix
L_0	reference evaluation distance for the mode mixity
L_e	length of beam finite element e
$M_{x,y,z}$	cross section moments around the x, y and z axis
n_b	number of beam elements in the beam finite element assembly
n_s	number of beam elements in the solid finite element assembly
$\hat{\mathbf{r}}_e$	element internal reaction forces at beam finite element e
$\overline{\mathbf{r}}_e$	element internal reaction forces at 3D solid finite element e
$\mathbf{r}_e$	element internal reaction forces at cross section finite element e
r	distance from the crack tip
$r_i^{1,2,3}$	nodal force at the crack tip in the x_1, x_2 and x_3 direction at node i
$\mathbf{s}$	total displacement of a point in the beam cross section
$T_{x,y,z}$	cross section forces in the x, y and z directions of the cross section coordinate system
$\mathbf{u}$	displacement of a point in the beam cross section due to warping deformation
$\hat{\mathbf{u}}, \hat{\mathbf{u}}_e$	global and element displacement vector for beam finite element model
$\overline{\mathbf{u}}, \overline{\mathbf{u}}_e$	global and element displacement vector for 3D solid finite element model
$u_i^{1,2,3}$	nodal displacement at node i in the x_1, x_2 and x_3 directions of the crack coordinate system
$\Delta u_{i,j}^{1,2,3}$	relative nodal displacements between node i and j in the x_1, x_2 and x_3 directions of the crack coordinate system
$\mathbf{v}$	displacement of a point in the beam cross section due to rigid body motion
$\mathbf{w}, \mathbf{w}_e$	global and element displacement vector for cross section finite element model
W	width of the square cross section
x, y, z	axes of the cross section coordinate system
X, Y, Z	axes of the global coordinate system
x_1, x_2, x_3	axes of the crack coordinate system
β	second Dundurs parameter
η	bi-material constant
$\boldsymbol{\theta}$	cross section forces and moments
$\kappa_{x,y,z}$	cross section curvatures around the x, y and z directions of the cross section coordinate system
κ_α	generalized plane stress material parameter of material α
μ_α	shear modulus of material α
ν_α	Poisson's ratio of material α
$\tau_{x,y,z}$	cross section shear strains in the xz (τ_x) and yz (τ_y) planes, and normal strain in the z direction (τ_z) according to the cross section coordinate system
ψ	mode mixity angle
$\boldsymbol{\psi}$	cross section strains and curvatures
$\hat{()}$	quantities associated with the beam finite element model
$\overline{()}$	quantities associated with the 3D solid finite element model
BECAS	BEam Cross section Analysis Software
ERR	energy release rate
FE	finite element
VCCT	Virtual Crack Closure Technique

In this paper an efficient framework is proposed for the computation of all three components of the ERR. The method is applicable to beam structures featuring a crack that extends along the length of the structure, as shown in Fig. 1. In engineering practice a crack that does not extend along the entire length of the beam, can often conservatively be assumed to do so.