Engineering Fracture Mechanics 133 (2015) 163–178

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

A fully automatic polygon scaled boundary finite element method for modelling crack propagation

Shangqiu Dai [a,*], Charles Augarde [b], Chengbin Du [a], Denghong Chen [c]

[a] Department of Engineering Mechanics, Hohai University, Nanjing 210098, China
[b] School of Engineering and Computing Sciences, Durham University, Durham DH1 3LE, UK
[c] College of Civil Engineering and Architecture, China Three Gorges University, Yichang 443002, China

ARTICLE INFO

Article history:
Received 10 July 2014
Received in revised form 28 October 2014
Accepted 23 November 2014
Available online 27 November 2014

Keywords:
Scaled boundary finite element method
Crack propagation
Fracture
Polygon elements

ABSTRACT

An automatic crack propagation remeshing procedure using the polygon scaled boundary FEM is presented. The remeshing algorithm, developed to model any arbitrary shape, is simple yet flexible because only minimal changes are made to the global mesh in each step. Fewer polygon elements are used to predict the final crack path with the algorithm as compared to previous approaches. Two simple polygon optimisation methods which enable the remeshing procedure to model crack propagation more stably are implemented. Four crack propagation benchmarks are modelled to validate the developed method and demonstrate its salient features.

1. Introduction

Fracture in brittle materials is an important issue in structural damage and failure. Modelling mixed-mode crack propagation in this material, combined with linear elastic fracture mechanics (LEFM), is an active research field and considerable efforts have been made in recent years. Studies are usually implemented within four broad frameworks, i.e. the finite element method (FEM), the boundary element method (BEM) and two more recent numerical approaches: meshless methods and extended FE methods.

Since the FEM was first used to model crack propagation in reinforced concrete beams by Ngo and Scordelis [1], it has been the predominant numerical method for crack problems [2–4]. However, crack propagation modelling with the FEM is still a challenging subject, because it usually requires both fine crack tip meshes [5,6] and sophisticated remeshing algorithms during crack propagation [7,8] to accurately calculate fracture parameters such as stress intensity factors (SIFs). Various methods have been proposed to extend the application of the FEM, such as the superposition method [9], quarter-point elements [10], and hybrid crack elements [11]. The extended FEM (XFEM) [12] and meshless methods [13] using nodal enrichment techniques, which can model crack propagation without remeshing, have been proposed more recently as alternatives to tackle the difficulties faced by the FEM in crack propagation modelling. Serious crack propagation problems both in statics and dynamics have been solved with these two methods [14–17]. However, fine meshes (in the case of XFEM) and fine nodal distributions (in the case of meshless methods), are still needed, especially when the crack paths are unknown in advance [18].

The BEM is a competitive alternative to FEM in crack propagation modelling because only the boundaries of the problem domain are discretised to define the geometry and only new boundary elements need to be added to the crack tip during

* Corresponding author. Tel.: +86 07821 287206.
E-mail address: hhudsq@gmail.com (S. Dai).

http://dx.doi.org/10.1016/j.engfracmech.2014.11.011
0013-7944/

Nomenclature

B^1, B^2	linear operator matrices of the SBFEM system
c	integration constants of the SBFEM system
D	elasticity matrix
E^0, E^1, E^2	coefficient matrices of the SBFEM system
K	global stiffness matrix
K_{si}	stiffness matrix of one subdomain
K_I, K_{II}	model-I and model-II stress intensity factors
L_0	distance between the crack tip and the boundary
N	shape function
P	equivalent nodal force
P_{si}	equivalent nodal force of one subdomain
u	displacement field
Δa	crack increment length
ξ, s	local coordinate system of the SBFEM system
r, θ	polar coordinates
λ	diagonal matrix
φ	displacement vector of the SBFEM system

remeshing. Although the BEM has these appealing features, its requirement of fundamental solutions limits its applicability considerably, generally to linear problems. Enormous research effort has sought to improve the existing BEM approach, with new techniques evolving such as the dual boundary element method (DBEM) [19] and the dual reciprocity method [20], but more exciting progress seems likely with enriched BEM methods [21].

The scaled boundary finite element method (SBFEM), is a semi-analytical method and was first developed by Wolf and Song [22] in the 1990s. It not only combines the advantages of FEM and BEM, but also possesses key features to make crack modelling more effective and efficient, and also has useful attributes such as the ease with which accurate SIFs can be extracted directly from the semi-analytical solutions [23,24] without fine crack tip meshes or special elements. Due to these attributes, the SBFEM has been successfully coupled with the BEM [25] and XFEM [26,27] for calculating parameters in fracture mechanics. These advantages have also been exploited in modelling both static and dynamic crack propagation problems by developing a simple remeshing procedure based on LEFM [18,28]. The advantage of the remeshing procedure is more significant when combined with cohesive interface finite elements (CIEs), a method named the FEM–SBFEM coupled method [29–31], to model cracks in nonlinear fracture mechanics (NFM). However, for problems with arbitrary multiple cracks in any arbitrary domain, the remeshing method is a little cumbersome because the subdomains may become so distorted that not all the nodes are directly visible from the scaling centres (an issue discussed in detail later in this paper).

A more general implementation of the SBFEM, where subdomains are discretised with polygon elements, was proposed by Ooi et al. [32,33] to overcome the problems described above. Polygon elements permit meshing of complex geometries flexibly, and the method uses a very simple local remeshing algorithm which just makes minimal changes to the global mesh by modelling the crack from one subdomain into an adjacent subdomain. However, the polygon mesh has to be refined in order to calculate the crack path accurately, because the crack propagation length is determined by the average distance from the vertices of the cracked subdomain to its scaling centre. Although a robust remeshing technique [34,35] was later proposed by Song and colleagues, similar to the hybrid FEM–SBFEM [36,37], the background mesh in this method needs to be stored and updated throughout the entire simulation, which makes it potentially computationally inefficient.

Recently, the polygon SBFEM was proved to be more accurate when compared with the conventional polygon FEM and cell-based smoothed polygon FEM [38] and a scaled boundary polygon formulation was developed to model elasto-plastic material responses in structures [39], which increases the application possibilities of the polygon SBFEM.

This study extends the simple remeshing algorithm developed in [18,28] to model polygon scaled boundary elements in brittle materials with any arbitrary shape. This remeshing algorithm is augmented to enable the modelling of crack propagation more stably and accurately. This paper is organised as follows: Section 2 briefly discusses the polygon SBFEM and its procedures for computation of SIFs. Section 3 discusses the pre-processing module used to generate polygon elements based on open-source Delaunay tessellation software. Section 4 addresses the remeshing algorithm and its improvement applied to the polygon SBFEM. Section 5 demonstrates the application of the algorithm to three mixed-mode crack propagation examples and Section 6 summarises the major conclusions that can be drawn from this study.

2. Polygon scaled boundary finite element method

The basic concept of the SBFEM is illustrated in Fig. 1(a) and (b). Fig. 1(a) shows a typical domain of arbitrary shape modelled by four subdomains, in which subdomain 1 contains a crack (named the cracked subdomain). Each subdomain has a scaling centre, from which all the boundaries of this subdomain are visible. Only the boundaries of the subdomains are