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The simulation of inelastic matrix strains in cementitious materials using micromechanical solutions

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ABSTRACT

A new approach is described for simulating inelastic behaviour in the matrix component of a two-phase composite material. Quasi-isotropic distributed micro-cracking, accompanying volumetric matrix changes, is combined with anisotropic micro-cracking arising from directional loading. An exterior point Eshelby solution is used to obtain stress concentrations adjacent to inclusions. The accuracy of these solutions is assessed using a series of three dimensional finite element analyses. A set of stress/strain paths are considered to illustrate the model's characteristics. The model is then applied to the problem of autogenous shrinkage in a cementitious composite, giving results that compare favourably with experimental data.

1. Introduction

Micromechanical models allow individual material properties, micro-cracking and inelastic behaviour to be modelled at the particle scale of a composite material. They also provide a means of linking the predicted behaviour to the macro-scale response. This paper describes a model for a two-phase composite material which has a matrix phase and inclusions. The particular focus is on simulating inelastic behaviour in the matrix phase alone [1]. Inelastic strains may derive from shrinkage, creep, micro-cracking, differential thermal expansion or ageing. These time dependent phenomena are particularly important when simulating cementitious composite materials such as concrete.

Neville et al. [41] reviewed a number of two-phase models for creep and shrinkage of concrete, including those of Hirsch [27], Counto [14] and England [21], in which the behaviour of the composite was derived from the properties of the aggregate and cement paste phases. A number of more recent models are based on multi-level schemes in which macro-scale stresses and strains are derived by up-scaling the behaviour at the micro-scale and below. Xi and Jennings [56] presented a multi-scale model for shrinkage in concrete and in cement paste that considered the behaviour from the nano to the meso-scale. Bernard et al. [8] described the inelastic strains from chemical shrinkage in cementitious composites with a multi-level model and Pichler et al. [45], also using a multi-level scheme, simulated early age autogenous shrinkage for the same type of cement based material. The latter model was further developed to include up-scaling of creep properties [44].

A two level multi-staged model was presented by Scheiner et al. [47] to describe creep in concrete in which the creep in cement hydrates was considered explicitly. These multi-scale models are particularly successful at simulating the development of strength during cement hydration [42]. The latter model has recently been employed in a combined experimental–numerical investigation of the micro-structure of hardened cement paste (hcp) which explored the importance of the

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Nomenclature

$\mathbf{A}_{\Omega}$	as defined for Eq. (3)
$\mathbf{A}_{\Omega\omega_v}$	as defined in Eq. (14)
$A_{\Omega v}$	as defined in Eq. (40)
a	radius of the spherical inclusion
A_E	activation energy
$\mathbf{C}_{add}$	total added compliance
c_{β}	evolution constant
C_{cem}	cementitious material content
c_E	constant as defined for Eq. (A.6)
c_{f_c}	constant as defined for Eq. (A.8)
c_{f_t}	constant as defined for Eq. (A.9)
$\mathbf{C}_{LM}$	matrix elastic compliance
$\mathbf{C}_L$	elastic compliance
$\mathbf{D}_M$	matrix elastic tensor
$\mathbf{D}_{M\omega_v}$	volumetric micro-cracked matrix tensor
$\mathbf{D}_{M\Omega}$	composite elastic tensor
$\mathbf{D}_{M\Omega\omega_v}$	volumetric micro-cracked composite tensor
$\mathbf{D}_{\Omega}$	inclusion elastic tensor
$\mathbf{D}_{Sec}$	secant constitutive matrix
E_{Ω}	inclusion Young's modulus
E_d	composite Young's modulus
E_M	matrix Young's modulus
E_v	volumetric Young's modulus
f	crack density parameter
F_{ζ_d}	directional micro-cracking function
F_{ζ_v}	volumetric micro-cracking function
f_c	compressive strength
f_t	tensile strength
f_M	volume fraction matrix
f_{Ω}	volume fraction inclusion
f_{td}	local directional tensile strength at the aggregate/cement paste interface
f_{tv}	local volumetric tensile strength at the aggregate/cement paste interface
H_{cem}	heat of hydration for cement
H_{FA}	heat of hydration for fly ash
H_{slag}	heat of hydration for slag
H_{uls}	ultimate heat of hydration
H_u	total heat of hydration
h_d	3 times the size of coarse aggregate
h_v	size of the coarse aggregate
$\mathbf{I}^{4s}$	fourth order identity tensor
i	integration direction
K_M	bulk modulus of matrix
$K_{M\Omega v}$	bulk modulus of composite as a function of solidification
K_{Mv}	bulk modulus of matrix as a function of solidification
K_{Ω}	bulk modulus of inclusion
$\mathbf{N}$	stress transformation tensor
$\mathbf{N}_{\varepsilon}$	strain transformation tensor
n_i	total number of integration directions
p_{cem}	total cement fraction
p_j	fraction by weight of cement
$\boldsymbol{r}, \boldsymbol{s}, \boldsymbol{t}$	local coordinate system
R	universal gas constant
r_{ζ_d}	as defined for Eq. (30)
$\mathbf{S}_E(\mathbf{x})$	exterior point Eshelby tensor
$\mathbf{S}_{\Omega}$	interior point fourth order Eshelby tensor
S_{Ω}	volumetric interior point Eshelby scalar
s_I	local principal stress
$s_{M\Omega}$	transformed amplified stress adjacent to inclusion
Δt	time step interval