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Computation of dynamic stress intensity factors in cracked functionally graded materials using scaled boundary polygons

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ABSTRACT

In this paper, the recently developed scaled boundary polygons formulation for the evaluation of stress intensity factors in functionally graded materials is extended to elasto-dynamics. In this approach, the domain is discretized using polygons with arbitrary number of sides. Within each polygon, the scaled boundary polygon shape functions are used to interpolate the displacement field. For uncracked polygons, these shape functions are linearly complete. In a cracked polygon, the shape functions analytically model the stress singularity at the crack tip. Therefore, accurate dynamic stress intensity factors can be computed directly from their definitions. Only a single polygon is necessary to accurately compute the stress intensity factors. To model the material heterogeneity in functionally graded materials, the material gradients are approximated locally in each polygon using polynomial functions. This leads to semi-analytical expressions for both the stiffness and the mass matrices, which can be integrated straightforwardly. The versatility of the developed formulation is demonstrated by modeling five numerical examples involving cracked functionally graded specimens subjected to dynamic loads.

1. Introduction

Functionally graded materials (FGMs) are a new class of customizable material arising from advancements in materials science and technology. Heat, oxidization, and fracture resistances can be increased by tailoring the volume fractions of the individual material constituents in a predetermined profile. This allows engineers to optimize the performance of FGM components under critical and extreme conditions for safety–critical structures in key modern industries such as microelectronics, aerospace and nuclear energy.

As the application of FGMs become more common in engineering practice, the extent to which these materials can be tailored against damage becomes more important. Therefore, the capability to analyze the fracture behavior of structures and components made of FGMs is crucial to their design and development. For this purpose, both static and dynamic analyses can be used. While static analyses provide engineers and designers with an indication of the critical state of stress in a cracked body, real world structures are invariably loaded dynamically. As such, there is an emphasis for the accurate analyses of fractured FGM structures and components that are subjected to dynamic loads.

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Nomenclature

a	crack length
$\mathbf{B}$	strain–displacement matrix
c_d	dilatational wave speed
$\mathbf{D}$	constitutive material matrix
ϵ	strain
η	circumferential coordinate
$\mathbf{E}_i$	coefficient matrices
E	Young's modulus
$\mathbf{F}$	force vector
$\mathbf{I}$	identity matrix
$\mathbf{J}(\eta)$	Jacobian matrix
$\mathbf{K}(\theta)$	vector of generalized stress intensity factors
$\mathbf{K}$	stiffness matrix
L	characteristic length
$\mathbf{M}$	mass matrix
$\mathbf{N}$	circumferential shape function matrix
Ω	domain
$\mathbf{\Phi}$	polygon shape function matrix
$\mathbf{\Psi}$	Schur transformation matrix
$\mathbf{\Psi}_\epsilon$	strain modes
$\mathbf{\Psi}_{\sigma L}^{(s)}$	singular stress modes at characteristic length
$\mathbf{\Psi}_\sigma$	stress modes
$\mathbf{\Psi}_\sigma^{(s)}$	singular stress modes
$\mathbf{\Psi}_u$	displacement modes
ρ	mass density
$\mathbf{S}_n$	block diagonal Schur matrix
σ	stress
Δt	time step
t	time
$\ddot{\mathbf{u}}_b$	vector of nodal acceleration
$\dot{\mathbf{u}}_b$	vector of nodal velocity
$\mathbf{u}_b$	vector of nodal displacements
ν	Poisson's ratio
ξ	radial coordinate
$\mathbf{Z}$	Hamiltonian coefficient matrix

The stress intensity factors (SIFs) play an important role in characterizing the fracture of FGMs. To compute the SIFs, both analytical and numerical methods can be used. The analytical solutions of the dynamic SIFs in FGMs have been reported by many researchers e.g. [1–10]. These studies are, however, limited to a finite crack in an infinite medium subjected to simple load cases. Although analytical methods provide direct closed-form solutions, the trade-off is a loss of generality and the need for complex solution techniques. Alternatively, numerical methods are more versatile and can be used to model a wider class of problems.

The finite element method (FEM) is the most widely used numerical method for structural analysis. When standard FEM procedures are applied to fracture analyses, a very fine mesh is required at the crack tip in order to obtain reliable results. Otherwise, special elements such as the quarter-point element [11], or elements with embedded asymptotic expansions [12], have to be used. Moreover, the FEM requires additional post-processing methods such as the *J*-integral [13] or the *M*-integral [14]. These techniques need to be specifically formulated for nonhomogeneous materials such as FGMs. Wu et al. [15] reported the formulation of the *J*-integral for dynamic fracture analysis of FGMs. Song and Paulino [16] derived the *M*-integral for the dynamic response of FGMs, and found the method to be superior to the *J*-integral and the direct correlation technique in calculating the dynamic SIFs.

The extended finite element method (XFEM) resolves some of the issues of mesh refinement about the crack tip in the FEM by enriching the shape functions of the finite elements in the vicinity of the crack with asymptotic crack tip functions [17]. Application of the XFEM to compute the dynamic SIFs in FGMs has been reported by Motamedi and Mohammadi [18], Singh et al. [19], Bayesteh and Mohammadi [20] and Liu et al. [21]. To compute the stress intensity factors, additional postprocessing techniques e.g. the *J*- and *M*-integrals are still required.