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Subsurface crack problems in a cubic piezoelectric material

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ABSTRACT

Subsurface crack problems in a cubic piezoelectric material are considered. The problems are formulated as singular integral equations and solved by a direct numerical method. Among normal and parallel cracks, only the parallel cracks with constant electric potential boundary condition imposed on the half-space that permeable and impermeable cracks need to be differentiated. The numerical accuracy of the present results is confirmed by comparing them with known exact solutions. The influence of material properties on the dimensionless electromechanical field intensity factors is found to be through an electro-mechanical coupling factor introduced in the text.

1. Introduction

In order to understand the fracture behavior of piezoelectric materials, crack problems in piezoelectric materials have been intensively studied experimentally and theoretically. There is now a vast body of literature on this topic and interested readers may find numerous articles referenced in [1–8]. It should be pointed out that most numerical results have been reported in isolation for materials with hexagonal symmetry. How these results are relevant to materials of different symmetry class is uncertain. As the influence of the material properties cannot be fully revealed through those studies, even for materials of the same class have to be investigated case by case. To widen our horizon an attempt has been made to study defect-related problems in cubic piezoelectric crystals [9–13]. This follow-up article addresses some subsurface crack problems in cubic piezoelectric materials by modeling the crack as a continuous distribution of dislocations; and such an approach leads naturally to singular integral equations with Cauchy type kernel. The traditional approach to solving singular integral equations is to reduce them to Fredholm integral equations [14] and then followed by an analytical method (say successive approximation or Eigen-function expansion) or a numerical treatment. The mathematical steps involved tend to be tedious and complicated however. A simplified approach is adopted here by directly converting the singular integral equations into their corresponding algebraic equations through appropriate integration formulas [15]. And this method has proven quite successful in solving a variety of crack problems in the past [16–21].

There are of course other methods can be used to solve crack problems. But, the application of dislocation methods to crack problems provides a unique perspective on looking at the fracture behavior of materials; not only elastic cracks but also elastoplastic cracks can be treated in a unified framework [22,23].

In the next section, the general solution of a cubic piezoelectric material subjected to an anti-plane deformation and in-plane electric field in terms of two complex functions are reviewed. Particularly, the near-crack-tip displacement discontinuity and electric potential jump across the crack surfaces are shown to be connected to the stress intensity factor and

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Nomenclature

a depth of the upper tip of a subsurface normal crack
b depth of the lower tip of a subsurface normal crack
b_s Burgers vector of a screw dislocation
b_ϕ electrical potential jump associated with a dislocation
c half crack-length
C_{11}, C_{12}, C_{44} elastic constants
d depth of a subsurface parallel crack
D_x D_y electric displacement components
D^A uniformly applied electric load on the crack surface
e_{14}, e_{15} piezoelectric constant
E_x E_y electric field components
h electromechanical coupling factor (defined as $e_{14}^2/(C_{44}\kappa_{11})$)
k electromechanical coupling factor (defined as $e_{15}^2/(C_{44}\kappa_{11})$)
K_{III} stress intensity factor of mode III
$\overline{K}_{III}$ dimensionless stress intensity factor (defined in Eqs. (39) and (42) for pure mechanical loadings and pure electric loadings respectively)
K_D electric displacement intensity factor
$\overline{K}_D$ dimensionless electric displacement intensity factor (defined in Eqs. (40) and (41) for pure electric loadings and pure mechanical loadings respectively)
s_1, s_2 real parameters (equal to $\sqrt{1+h}+\sqrt{h}$ and $\sqrt{1+h}-\sqrt{h}$ respectively)
w displacement
x y rectangular coordinates
z_1 z_2 complex variables (defined as $x+\mu_1 y$ and $x+\mu_2 y$ respectively)
γ_{yz} γ_{zx} shear strain components
Φ electric potential
κ_{11} dielectric constant
λ Real parameter defined as $-\sqrt{C_{44}/\kappa_{11}}$
λ_1, λ_2 pure complex parameters (equal to $i\lambda$ and $-i\lambda$ respectively)
μ_1, μ_2 pure complex parameters (equal to is_1, is_2 respectively)
τ^A uniformly applied shear stress on the crack surfaces
τ_{yz} τ_{zx} shear stress components

electric displacement intensity factor respectively. This allows one to determine these two fracture parameters once the near-crack-tip displacement discontinuity and electric potential gap are known. In Section 3, the fundamental solution of a subsurface dislocation with electric potential gap is presented. Similar to [12], two types of electric boundary condition on the half-space surface are considered. Based on this fundamental solution, singular integral equations are formulated for the crack problems by simulating the crack as a continuous distribution of dislocations in Section 4. The numerical method adopted in this article is described in Section 5 and the results are reported and discussed in Section 6. Since results of parallel permeable crack problems have been reported and discussed earlier [12]; here the focus of our attention is on normal and parallel impermeable cracks. In fact as shown later, only under the boundary condition of imposing constant electric potential on the surface of the half-space, that permeable and impermeable cracks need to be differentiated for subsurface parallel cracks. Conclusions are drawn in the last section.

2. Basic equations

A rectangular coordinate system is chosen with the x-axis being along [100], the y-axis along [010] and the z-axis along [001] of the cubic crystal. For cubic crystals, the electro-elastic coupling can be separated into two types: (i) in-plane deformation coupled to out-of-plane electric field, or

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ D_z \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 \\ C_{12} & C_{11} & 0 & 0 \\ 0 & 0 & C_{44} & -e_{14} \\ 0 & 0 & e_{14} & \kappa_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ E_z \end{Bmatrix} \tag{1}$$

and (ii) out-of-plane deformation coupled to in-plane electric field, or