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A hybrid crack tip element containing a strip-yield crack-tip plasticity model

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ABSTRACT

This paper presents a model for the simulation of cracks in thin-walled structures using Trefftz-elements (T-elements) for crack tip regions. These elements are based on exact solutions to the governing differential equations and inner boundary conditions, resulting in high resolutions of the stress/strain fields. The T-elements are combined with standard elements using an extended variational principle. Beside this formalism the paper presents the derivation of particular solutions for a straight 2d-crack including a Dugdale strip-yield zone. The results are validated against high-resolution standard finite element simulations. Finally a crack propagation algorithm and some future enhancements are discussed.

1. Introduction

The explicit Finite Element Method (FEM) is established as the standard simulation tool in the field of mechanical engineering. Although this method provides a high prediction quality concerning deformations and intrusions, strongly localized phenomena like failure of joints, crack initiation and propagation, etc. cannot be described sufficiently without locally fine re-meshing. As a result of the Courant–Friedrich–Levy criterion for the critical simulation-time-step the elements have to be kept of specific minimum sizes to avoid escalating computational times. One example where rather coarse meshes are used is the simulation of car crashes. The mesh dependency of the achievable resolution in stress/strain fields as well as the inability to describe continuous crack propagation are considerable limitations of the standard FEM for the simulation of crack propagation. Due to these problems many different numerical approaches were developed. A review of computational methods for fracture in brittle and quasi-brittle solids can be found in [1]. From the several methods used today, e.g. meshless methods, cracking particles method or isogeometric analysis, here will be described only two commonly used approaches.

The extended finite element method (XFEM) was introduced in [2] and has become very popular in fracture mechanics; see [3] for a review of this method. In this methods elements near the crack are enriched by additional ansatz functions. The enrichment is realized through the partition of unity concept. Elements cut by the crack are enriched with Heaviside functions, which allow a discontinuity in the displacement within an element. In this way the crack path does not need to align with the FE mesh. Elements near the crack tip are ideally enriched with parts of the analytic solution of the problem, e.g. for brittle fracture the analytical solution of [4] is widely used with XFEM. The use of the analytic solution results in higher accuracy then the standard FEM and thus a coarser mesh can be used for XFEM (this is not the case when the enrichment differs from the analytical solution). A method for tracking the crack can be chosen independently from the XFEM method, common

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Nomenclature

A_k	coefficients for homogeneous part of ϕ
a_n	coefficients for particular part of ϕ
c	physical crack length of the edge cracked panel
f	conformal map
h	height of the edge cracked panel
$\boldsymbol{K}$	Trefftz-element nodal stiffness matrix
n	complex representation of the normal unit vector
$\boldsymbol{p}$	Trefftz-element nodal load vector
q	complex displacement
$q_h,\ q_p$	homogeneous and particular part of the complex displacement
$\hat{q}$	prescribed displacement on Γ_0
r_D	length of the Dugdale-zone
t	plate thickness
w	width of the edge cracked panel
$\widehat{W}_n$	ansatz functions for particular parts of ϕ
$\phi,\ \psi$	complex holomorphic potentials
Γ_0	boundary of essential boundary conditions
Γ_1	boundary of force boundary conditions
Γ_2	boundary between crack tip region and uncritical domain
$\kappa,\ \lambda,\ \mu$	Lame constants
ν	Poisson ratio
$\sigma_{xx},\ \sigma_{yy},\ \tau_{xy}$	plane stress components
$\sigma_h,\ \sigma_d$	homogeneous and deviatoric stress components
σ_y	yield stress
τ	complex traction
$\tau_h,\ \tau_p$	homogeneous and particular part of the complex traction
$\hat{\tau}$	prescribed traction on Γ_1
$\hat{\tau}_D$	crack closing complex traction
$\Omega = \Omega_1 \cup \Omega_0$	computational domain
Ω_1	crack tip region
Ω_0	uncritical domain

choices are crack representations as straight line segments or using level sets (these can also allow curved cracks), compare [5]. Between the standard FE elements and elements enriched with non-Heaviside functions blending elements are needed to avoid unwanted terms at the boundary between enriched and not enriched elements, see [6]. Special care needs to be taken for the integration of the enriched elements. Sub-triangulation of elements cut by the crack or containing the crack tip is needed together with an adaption of the degree of the quadrature rule. Especially in 3d this is a major task. The XFEM approach is well suited for a small number of cracks while the handling of many cracks becomes cumbersome.

Another way to avoid mesh-refinement in standard FEM is the use of special purpose elements, which are better adapted to the local conditions at the crack tip than conventional finite elements, based on polynomial shape functions. This paper suggests the use of so called hybrid Trefftz-elements (T-elements) for the crack tip region, which stems from works of [7–10]. In contrast to conventional finite elements, a T-element uses particular solutions of the governing differential equations, which also satisfy inner boundary conditions [11,12]. This approach results in high resolutions of the stress/strain fields in the vicinity of the crack tip without a mesh-refinement in the critical region. In [13] a hybrid crack element with p-adaptivity is introduced for Mode I linear elastic crack tip, i.e. higher order terms of the analytical solution [4], were considered. This approach was then extended to mixed mode problems in [14]. It turned out that the coefficients of the higher order terms are harder to determine than the stress intensity factor which is the lowest order coefficient. This p-adaptive hybrid element was compared to an XFEM setting were the same higher order terms were used for the enrichment in [15]. It was stated that the accuracy of the higher order coefficients calculated with the XFEM approach was low, which might be caused by a low order quadrature rule. A general advantage of hybrid Trefftz elements is that they allow the construction of accurate crack-singular elements using the known local analytical solution but no fundamental solution is needed as e.g. in the boundary element method. Additionally, using the hybrid Trefftz method a quasi continuous crack growth can be realized.

For the simulation of cracks in quasi-brittle materials strip-yield models, introduced by Baerenblatt [16] and Dugdale [17], are widely used in engineering applications. These models describe the plasticity effects of the material without the need of nonlinear material models. The crack is virtually extended with the strip-yield zone, where a non-vanishing traction acts. This traction represents the hardening and softening of the material. The original crack remains traction free. Aspects of