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Element-wise fracture algorithm based on rotation of edges

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ABSTRACT

We propose an alternative, simpler algorithm for FEM-based computational fracture in brittle, quasi-brittle and ductile materials based on edge rotations. Rotation axes are the crack front edges (respectively nodes in surface discretizations) and each rotated edge affects the position of only one or two nodes. Modified positions of the entities minimize the difference between the predicted crack path (which depends on the specific propagation theory in use) and the edge or face orientation. The construction of all many-to-many relations between geometrical entities in a finite element code motivates operations on existing entities retaining most of the relations, in contrast with remeshing (even tip remeshing) and enrichment which alter the structure of the relations and introduce additional entities to the relation graph (in the case of XFEM, enriched elements which can be significantly different than classical FEM elements and still pose challenges for ductile fracture or large amplitude sliding). In this sense, the proposed solution has algorithmic and generality advantages. The propagation algorithm is simpler than the aforementioned alternatives and the approach is independent of the underlying element used for discretization. For history-dependent materials, there are still some transfer of relevant quantities between meshes. However, diffusion of results is more limited than with tip or full remeshing. To illustrate the advantages of our approach, two prototype models are used: tip energy dissipation (LEFM) and cohesive-zone approaches. The Sutton crack path criterion is employed. Traditional fracture benchmarks and newly proposed verification tests are solved. These were found to be very good in terms of crack path and load/deflection accuracy.

1. Introduction

Discretization methods for computational fracture can be performed with meshfree (cf. [52–55,64–68]) and finite elements (cf. [5,27]). In the former, crack propagation algorithms have been developed in the past two decades with varying degrees of effectiveness and generality. Existing techniques can be classified as discrete or continuum-based (including combinations of these). A non-exhaustive list is:

- Full and localized rezoning and remeshing approaches [21,31,9,14], variants of local displacement [47,46,39,43] (or strain [49,2]) enrichments, clique overlaps [29,38], edges repositioning or edge-based fracture with R-adaptivity [45].

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Nomenclature

a	crack closure parameter
$\boldsymbol{b}_\star$	nominal body force
d	damage variable for the cohesive law
$\boldsymbol{e}$	external forces
$\hat{\boldsymbol{e}}_I$, $\hat{\boldsymbol{e}}_{II}$, $\hat{\boldsymbol{e}}_{III}$	Modes *I*, *II* and *III* opening directions (normalized)
f_t	normal stress
$f_I^+, f_{II}^+, f_{III}^+$	internal forces of modes *I*, *II* and *III*
$\boldsymbol{f}_{0i}^+$	internal force vector at the tip
$\dot{F}$	external force power
$\boldsymbol{i}$	internal forces
$\boldsymbol{I}$	identity matrix
J	strain energy release rate
J_R	LEFM fracture energy
$\boldsymbol{K}$	stiffness matrix
$\boldsymbol{l}$	derivative of the constraint equation
$\boldsymbol{n}$	normal to the plane or the shell surface
$\boldsymbol{p}$	crack path direction
Q	load proportionality factor
$\dot{q}$	velocity proportionality factor
$\boldsymbol{r}$	residual
s	area of Γ_{c_a}
s_c	constraint equation
$\dot{S}$	cohesive force power
t	time
$\boldsymbol{t}_\star$	imposed surface load
$\boldsymbol{t}_u$	reactive surface vector
$\boldsymbol{t}$	cohesive traction for the quasi-brittle case
$\boldsymbol{t}_\lambda$	cohesive traction for the brittle case
T	total time of analysis
$\boldsymbol{T}_{\text{mode}}$	transformation matrix at the tip
u_t	modes *II* and *III* equivalent displacement
u_I, u_{II}, u_{III}	displacement components of modes *I*, *II* and *III*
$\boldsymbol{u}$	displacement vector field
$\boldsymbol{u}_\star$	imposed displacement
$[[\boldsymbol{u}]]$	displacement jump
$[[\dot{\boldsymbol{u}}]]$	virtual velocity jump
$\dot{W}$	strain power
W_p	plastic work
$\boldsymbol{x}_0$	tip coordinates
$\boldsymbol{x}_1$, $\boldsymbol{x}_2$	coordinates of the two neighbor (opposing) nodes connected by external edges to the tip
β	modes *II* and *III* parameter
Γ_c	crack surface in the deformed configuration
Γ_{c_a}	active crack surface, $\Gamma_{c_a} \subset \Gamma_c$
Δs	increment of s
$\dot{\boldsymbol{\varepsilon}}$	strain rate
$\dot{\boldsymbol{\varepsilon}}_p$	plastic strain rate
θ_c	crack path angle
κ	kinematical variable for the cohesive law
κ_0	initial κ
σ	normal cohesive stress
$\boldsymbol{\sigma}$	Cauchy stress tensor
τ_{II}, τ_{III}	tangential cohesive stress components for modes *II* and *III*
ϕ	damage loading function
Ω	body deformed configuration integration domain