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Localisation analysis in masonry using transformation field analysis

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ABSTRACT

Transformation field analysis is used for the multi-scale analysis of cracking localisation in masonry. The evaluation of the corresponding consistent homogenised tangent stiffness required in acoustic tensor-based localisation analysis is derived. The average representative volume element mechanical response is studied for two sets of micro-mechanical material laws for mortar joints, based on damage and damage coupled with plasticity. Localisation analyses for stress proportional loading show that meaningful average localisation orientations are properly detected by the acoustic tensor-based loss of ellipticity criterion. The ability of the procedure to reproduce the failure envelope of running bond masonry subjected to uniform biaxial loads is demonstrated. The energetic aspects for the selection of localised solutions are discussed. It is shown that transformation field analysis yields meaningful homogenised localisation results, thereby allowing envisioning its use in nested multi-scale computations.

1. Introduction

Due to the complexity and the heterogeneous nature of masonry, the design of masonry structures is essentially based on codes and rules of thumb. In this respect, modern computational methods started to emerge as valuable complements for the study of masonry structures, see for instance [1–3]. The interest in such modelling tools originates from their rational approach towards stability studies, allowing complementing the codes and the experience of the analysts. However, these computational approaches require reliable constitutive models which are difficult to formulate for masonry.

Structurally, masonry may be considered as a two-phase composite material in which bricks and mortar are assembled often in a periodic manner. This periodic mesostructure and the different elastic characteristics of its constituents render the elastic behaviour of masonry anisotropic. In addition, the weak mortar phase with the periodic arrangement leads to a stiffness degradation along preferential orientations.

As a result, the formulation of constitutive models for structural scale computations is rather delicate. Such models link average stresses to average strains and can be based on plasticity concepts [4] or damage with stiffness degradation [5]. However, phenomenological laws are often based on assumptions that are difficult to validate, and their experimental identification and quantification is usually difficult and expensive.

Such drawbacks may be partially avoided by using multi-scale approaches, where the mesoscopic and macroscopic scales of representation are intrinsically linked. Closed-form constitutive relations are then postulated at the scale of constituents,

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Nomenclature

A	acoustic tensor
$\overline{\mathbf{A}}_r$	elastic mechanical concentration tensor of the sub-domain r
$\overline{\mathcal{A}}_r$	rate form of the concentration tensor of the sub-domain r
c	cohesion of masonry joints
c_r	volume fraction of the sub-domain r in the RVE
$\mathbf{C}^r$	stiffness tensor of the sub-domain r
$\overline{\mathbf{C}}$	homogenised elastic stiffness tensor
$\overline{\mathbf{C}}_t$	homogenised tangent stiffness tensor
D	scalar damage variable
$\overline{\mathbf{D}}_r^s$	average mechanical concentration tensor of the sub-domain r associated to an inelastic eigenstrain on the sub-domain s
E	Young modulus
$\mathbf{E}$	macroscopic strain tensor
f_c	compressive mode-I strength of masonry joints
f_t	tensile mode-I strength of masonry joints
G	shear modulus
G_{ft}	mode-I fracture energy of masonry joints (pure damage model)
G_{cI}	mode-I fracture energy of masonry joints (damage-plastic model)
G_{cII}	mode-II fracture energy of masonry joints (damage-plastic model)
$\overline{\mathbf{L}}_r^s$	average inelastic mechanical concentration tensor of the sub-domain r due to an eigenstrain applied on the sub-domain s
${}^4\mathbf{L}_e$	tensorial notation of macroscopic elastic material stiffnesses for a closed-form model
${}^4\mathbf{L}_s$	tensorial notation of macroscopic secant material stiffnesses for a closed-form model
${}^4\mathbf{L}_t$	tensorial notation of macroscopic tangent material stiffnesses for a closed-form model
$\vec{m}$	strain discontinuity mode vector
$\vec{n}$	normal vector
$\overline{\mathbf{Q}}_r^s$	average inelastic mechanical concentration tensor of the sub-domain r due to an eigenstrain applied on the sub-domain s
$\vec{u}$	mesoscopic displacement field
V	volume of the RVE
V_r	volume of the sub-domain r
$\vec{x}$	mesoscopic position vector
x_1	in-plane coordinate axis parallel to the bed joint
x_2	in-plane coordinate axis perpendicular to the bed joint
δ	Kronecker symbol
$\boldsymbol{\epsilon}$	mesoscopic infinitesimal strain tensor
ϵ_{eq}	scalar equivalent strain
ϕ_{mc}	friction angle of masonry joints
ϕ_c	cap compression angle of masonry joints
κ	mesoscopic damage-driving parameter
μ	friction coefficient of masonry joints
ν	Poisson ratio
$\boldsymbol{\Theta}$	orientation of the normal to the average crack pattern
$\boldsymbol{\sigma}$	mesoscopic stress tensor
$\boldsymbol{\Sigma}$	average macroscopic stress tensor

and the structural scale response of the masonry material is obtained by averaging operations. Such averaging operations have been used for masonry based on asymptotic homogenisation principles [6,7], computational homogenisation [8–11], or other averaging schemes [12].

Multi-scale approaches can be used to bridge the constituents and structural scales in an uncoupled fashion by identifying macroscopic closed-form material parameters from mesoscopic models. The actual structural computations are in this case performed subsequently with the macroscopic model only as in [4]. However, in addition to this parameter identification issue, formulating an accurate closed-form macroscopic constitutive law itself constitutes a difficult task. Another use of multi-scale principles therefore consists of interrogating mesostructural samples during macroscopic computations in a nested procedure. Both scales of representation are then fully coupled in the entire structural computation, and the use of an explicit formulation of constitutive equations at the structural scale is avoided. Such nested approaches were mainly developed using computational homogenisation, first for discrete damage evolution laws in classical continua [13,14]. They were followed later by extensions of FE^2 strategies in which finite element approximations are used at both scales. A