Engineering Fracture Mechanics 110 (2013) 249–257

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

# Mode I stress intensity factor for cracked thin-walled open beams

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A R T I C L E I N F O

*Article history:*
Received 10 January 2013
Received in revised form 29 July 2013
Accepted 3 August 2013
Available online 14 August 2013

*Keywords:*
Cracked thin-walled beams
Stress intensity factor
Warping effect

A B S T R A C T

A general analytical method to determine the mode I stress intensity factor for thin-walled beams is presented. This method is based on the concept of crack surface widening energy release rate, which is expressed in terms of the $G^*$ integral and the thin-walled beam theory. A distinctive aspect of this technique is the incorporation of the warping effect, which is a common feature in thin-walled beams that significantly influences in the stress distribution. This characteristic gives generality to the method, allowing the analysis of crack scenarios that have not been yet considered by other authors. The results show a good agreement with shell finite element solutions and other results available in the literature.



## 1. Introduction

Thin-walled beams are widely employed in modern engineering structures. For this reason, the study of crack behavior in these structural components represents a topic of crucial importance. The mode I stress intensity factor is a very significant parameter in the integrity evaluation and risk analysis of structures. The determination of an exact solution for the stress intensity factor is usually a difficult enterprise. In thin-walled beams, the presence of sectional warping constitutes an additional problem. Although some approaches have recently been proposed in this direction [1–3], no one have regarded flexural–torsional loads, which activate the warping effect.

The purpose of this article is to present a technique to determine the mode I stress intensity factor for cracked thin-walled beams. This technique is based on the $G^*$ integral concept and the thin-walled beam theory. $G^*$ integral [4] is derived from the conservation law and the concept of crack mouth widening energy release rate. It has shown to be easy to employ in the determination of closed forms for the stress intensity factors in several crack problems [1,2,5]. Thin-walled open beam theory [6–8] considers the warping effect derived from the natural flexural–torsional coupling of this kind of structures. In the determination of mode I stress intensity factor, warping is taken into account by considering the energetic contribution of the bimomental force.

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0013-7944/$ - see front matter 
http://dx.doi.org/10.1016/j.engfracmech.2013.08.004

## Nomenclature

| | |
|---|---|
| $a$ | crack depth (also semi-major axis of the elliptic crack) |
| $\tilde{a}$ | boundary of the elliptic crack |
| $A$ | cross-sectional area |
| $b$ | dimension of a flange |
| $B$ | bimomental beam force (also point $B$, origin of the system $B$: $x$, $s$, $n$) |
| $c$ | semi-minor axis of the elliptic crack |
| $C$ | center of gravity of the uncracked cross section |
| $C_{ij}$ | element of order ($i$, $j$) of the inverse of the constitutive matrix |
| $C_w$ | warping constant |
| $G^*$ | crack mouth widening energy release rate |
| $\boldsymbol{G_c}$ | vector containing elements of the inverse of the constitutive matrix |
| $h$ | dimension of the web |
| $\boldsymbol{I_i}$ | identity matrix of size $i$ |
| $I_y$, $I_z$ | second moments of area |
| $I_{yz}$ | product moment of area |
| $I_{y\omega}$, $I_{z\omega}$ | product of warping |
| $\boldsymbol{J}$ | constitutive matrix |
| $K_I$ | mode I stress intensity factor |
| $L$ | length of the beam |
| $n$ | coordinate normal to the cross-section middle line |
| $N$ | axial beam force |
| $M_y$, $M_z$ | bending moments |
| $\boldsymbol{Q}$ | vector of generalized forces |
| $\boldsymbol{Q_c}$ | vector containing squares and products of the generalized forces |
| $r$ | radial coordinate |
| $s$ | circumferential coordinate |
| $S$ | cross-sectional perimeter |
| $S_y$, $S_z$ | first moments of area |
| $S_\omega$ | first moment of warping |
| $t$ | beam thickness |
| $\boldsymbol{T}$ | stress vector |
| $T_x$, $T_s$ | elements of the stress vector |
| $u$ | axial displacement |
| $\boldsymbol{u}$ | displacement vector |
| $U$ | strain energy |
| $U_0$ | strain energy density |
| $v$ | circumferential displacement |
| $x$, $y$, $z$ | Cartesian coordinates |
| $Y$, $Z$ | coordinates of a point located in the middle line of the cross-section |
| $\alpha_i$ | coefficients of the axial stress in cracked cross-section |
| $\gamma$ | vector of shape functions associated to cracked cross-section |
| $\Gamma$ | integration path |
| $\Delta$ | vector of generalized strains |
| $\eta_x$, $\eta_s$ | components of the unit outward normal vector |
| $\theta$ | angular coordinate |
| $\theta_x$ | warping variable |
| $\theta_y$, $\theta_z$ | bending twists |
| $\lambda$ | auxiliary integration variable |
| $\nu$ | Poisson's ratio |
| $\xi$ | crack location (axial coordinate) |
| $\Pi$ | total potential energy |
| $\sigma_{xx}$ | axial stress |
| $\omega_p$ | primary warping function |
| $(\cdot)^{(0)}$ | superscript associated to the uncracked cross-section |
| $(\cdot)^{(c)}$ | superscript associated to the cracked cross-section |
| $(\cdot)^{(R)}$ | superscript associated to the cracked flange or web of the beam |